Known unknowns: Epistemic inferences of superlative modifiers*

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Abstract This paper provides an investigation of the Ignorance Inferences that Superlative Modifiers (SMs) like *at least* and *at most* give rise to in non-embedding contexts. The formal properties of these IIs are characterized in terms of the epistemic conditions that they impose on the speaker, thereby establishing how much can and must be inferred about what the speaker is ignorant about. It is argued that the form of these IIs depends on the structural properties of the expression that the SM is modifying: SMs with totally ordered associates trigger IIs that are formally different than those of SMs with partially ordered associates. The paper presents a unified and comprehensive account of these IIs. The analysis proposed employs a neo-Gricean calculus with a double alternative generation mechanisms, one of which must be provided by focus alternative semantics. The results improve upon previous analyses by deriving the right form of IIs in the rich variety of environments in which SMs can appear.

Keywords: Ignorance inferences, implicatures, superlative modifiers, alternative semantics, focus association

1 Introduction

Superlative Modifiers (SMs henceforth) like *at most* and *at least* often convey that the speaker is uncertain or ignorant about something, and so she cannot commit to providing more information. This epistemic effect of SMs has been dubbed an Inference of Ignorance (II henceforth). As illustration, consider (1):

(1) a. #I have at most two daughters.
b. #I have at least five fingers.

* Comments are welcome! I am grateful to the people who provided comments and discussions at different stages of this paper: Athulya Aravind, Seth Cable, Ilaria Frana and Angelika Kratzer. Many thanks as well to the audiences at the venues where different parts of this project were presented: NELS 46, CLS 52, SALT 26 and especially the UMass Semantics Workshop. Athulya Aravind deserves special gratitude for her very helpful review of a previous draft. All errors are mine.

These examples are odd. The epistemic competence commonly assumed when we talk about progeny or our own body is at odds with the presence of SMs and their incompatibility with full knowledge. Contrast (1) with the felicitous (2), which differs only in that no speaker knowledge need be assumed.

(2) a. Bill has at most two daughters.
b. That caterpillar has at least twenty legs.

That SMs trigger IIs is uncontroversial. The controversy is about what exactly is the division of labor between semantics and pragmatics in deriving the various implications conveyed by sentences like (1). More specifically, it is debated whether IIs are encoded as part of the conventional meaning of SMs (Geurts & Nouwen 2007, Nouwen 2010) or whether they are the result of pragmatic processes (Büring 2007, Coppock & Brochhagen 2013b, Nouwen 2015, Schwarz 2016a, a.o.). To this end, a number of different proposals, each introducing its own machinery, have been put forward. On the semantic side, SMs have been analyzed as modals (Geurts & Nouwen 2007), as *minima* and *maxima* operators (Nouwen 2010), as inquisitive expressions (Coppock & Brochhagen 2013b), as operators of meta-speech acts (Cohen & Krifka 2014), and as epistemic indefinites (Nouwen 2015). On the pragmatic side, at least two main avenues of research have been pursued in deriving IIs: neo-Gricean analyses (Büring 2007, Schwarz 2016a) and those relying on grammatical approaches to implicatures (Mayr 2013).

Despite this attention that SMs have attracted recently, a careful investigation into the nature and form of their IIs has yet to be undertaken. That is, given a sentence containing an SM, how much *can* be inferred about what the speaker is ignorant about? Is there anything that speakers *must* be obligatorily ignorant about in order to successfully use an SM-expression? The lack of a formal description of these IIs in the variety of contexts in which they arise makes it difficult to evaluate theories of SMs for their empirical adequacy. This paper aims to fill this gap by offering a precise characterization of IIs with SMs and putting forward a theory of SMs that will capture those facts. I begin by sketching some desiderata that any satisfactory theory of SMs must meet.

1.1 Two questions

The first desiderata has to do with the following question: Is there anything that the speaker *must* be ignorant about in order to felicitously use an SM? Different formulations of IIs may make different predictions as to what inferences can be made about the speaker's epistemic state. For instance, consider the following sentences, taken from two recent accounts of SMs:

(3) a. At least three boys left. [Mayr 2013: 158]
b. Bill is at least an assistant professor. [Coppock & Brochhagen 2013b: 10]

According to Mayr, (3a) "seems to have the ignorance inference that for any number *n* larger than three, the speaker fails to believe that n-many boys left". Along similar lines, Coppock & Brochhagen suggest that the meaning of (3b) "can be expressed as a disjunction over the answers that are at least as strong: 'John is an assistant, associate, or full professor'. " Taking these descriptions at face value, one might be tempted to conclude that IIs of SMs should be understood as a list of disjunctive statements.

Alternatively, one might follow Büring (2007) and adopt a somewhat more restricted characterization of the sentences in (3), where the relevant IIs are about the disjunction of *three* and *more than three* in (3a), and about the disjunction of *assistant professor* and *a higher rank above assistant professor* in (3b).

(4) a. $[\![(3a)]\!] \rightsquigarrow$ *the speaker doesn't know whether exactly three or more than three boys left*
b. $[\![(3b)]\!] \rightsquigarrow$ *the speaker doesn't know whether John is an assistant professor or some higher rank*

The two characterizations may be summarized schematically as follows, using (3b) as an example:

(5) a. [*assistant professor*] or [*associate professor*] or [*full professor*]
b. [*assistant professor*] or [*more than an assistant professor*]

At first glance, the difference between (5a) and (5b) appears trivial. As will be demonstrated in this paper, the two views in fact make diverging predictions about the kind of inferences one is licensed to make about the speaker's ignorance. Whereas formulations like (5a) predict that the speaker believes that every rank above assistant professor is a possible rank for Bill, formulations like (5b) do not require that any particular rank above assistant professor is a mandatory epistemic possibility for the speaker, as long as one of them is. Thus, our first empirical goal should be to identify exactly what kinds of information are compatible with an utterance containing an SM. Call this question PREDICTABILITY:

(6) PREDICTABILITY
Is there any proposition in particular about which the speaker must be ignorant about so that she can successfully use an SM?

An additional layer of complexity is introduced by the fact that SMs may combine with a number of different types of complements or associates, consistently leading

to IIs in all these environments. As shown in (7)-(10), IIs of SMs might be present with a variety of scales (Hirschberg 1985).

(7) *Horn Scales*

a. At least some students came to the party.
⇝ the speaker is ignorant about whether all students came

b. At most a few students came to the party.
⇝ the speaker is ignorant about whether any student came

(8) *Cardinality Scales*

a. At least Bill came to the party.
⇝ the speaker is ignorant about whether someone else came to the party

b. At most Bill and Jane came to the party.
⇝ the speaker is ignorant about whether any of them came to the party

(9) *Lexical Scales*

a. Sue won at least the silver medal.
⇝ the speaker is ignorant about whether Sue won the golden medal

b. Sue won at most the silver medal.
⇝ the speaker is ignorant about whether Sue won any medal

(10) *Evaluative Scales* [for a preference scale ⟨ *broccoli, candy* ⟩]

a. Bill ate at least broccoli.
⇝ the speaker is ignorant about whether Bill ate candy

b. Bill ate at most broccoli.
⇝ the speaker is ignorant about whether Bill ate broccoli

Despite this flexibility of SMs, investigation into IIs has asymmetrically focused on the numeral case, where SMs modify numerals or measure phrases, leaving the cases involving associates of other categories (DPs, VPs, etc.) largely unexplored. A tacit assumption in the literature is that SMs behave alike in IIs across-the-board. However, the fact that potential associates of SMs show a rich variety of formal properties raises the possibility of non-uniformity in the nature of their IIs. Thus, a second empirical goal in this paper is to identify the precise nature of IIs across these various environments. Call this second question UNIFORMITY:

(11) UNIFORMITY
Are the inferences that come with SMs the same across the board, regardless of the associate type of the SM?

1.2 The plan

To summarize, a full characterization of IIs with SMs across environments is lacking, and therefore it is presently unclear whether extant theories of SMs are adequate in their empirical coverage. By answering the aforementioned questions about the PREDICTABILITY and UNIFORMITY of SMs' IIs we provide further criteria for evaluating theories of SMs. In the first part of this paper, I attempt to make precise the exact form of IIs with SMs and what they tell us about the speaker's epistemic state, thereby providing an answer to these questions. The analytic strategy I adopt is to compare IIs with SMs with those that arise with parallel sentences with disjunction. Two key empirical points emerge from this investigation: (i) ignorance with SMs may be partial (i.e. (5b) above is a better characterization of the speaker's epistemic state), and (ii) the nature of IIs is not uniform across associate types.

These descriptive findings will be used in the second part to provide a unified account for IIs. The specific account I will endorse takes SMs to be scalar modifiers interpreted relative to some focalized constituent. IIs with SMs arise as a kind of Quantity Implicature, derived in a neo-Gricean fashion. The calculation of implicatures with SMs will require two sets of alternatives, as has already been proposed by Mayr (2013), Kennedy (2015) and Schwarz (2016a). The main innovation of the calculus presented here is that each set of alternatives relevant for the Gricean computation is provided by a different, independent, mechanism. The first method is the familiar substitution method within elements of a Horn scale (Horn 1972, Sauerland 2004b, a.o.). Crucially, I take SMs to form a Horn scale with *only*, given the parallels between the two elements in terms of focus association (see also Schwarz 2016a). In addition, a different set of alternatives is obtained by replacing the focus-bearing constituent, i.e., the SMs' associate, with contextually relevant alternatives (Rooth 1992, Fox & Katzir 2011).

The remainder of this paper is organized as follows. I will first illustrate that SMs are compatible with partial ignorance and outline what the speaker must be ignorant about across different uses of SMs. I then motivate two key assumptions that will be crucial to my analysis of IIs with SMs–(i) that ignorance is pragmatic, and (ii) that SMs are conventionally associated with focus–before turning to the analysis itself in section §4. In section §5 the results obtained are assessed–including open questions an unsolved problems–and compared to other approaches. Section §6 provides a brief summary.

2 Characterizing Ignorance

The main goal of this section is to provide a proper characterization of IIs that SMs convey, by scrutinizing our intuitions about them. The literature offers two views about what the IIs of SMs should look like. For instance, for a sentence like (12) we find the two characterizations in (13):

(12) At least four people came to the party.

(13) a. OPTION 1: For any number $n \geq 4$, the speaker is ignorant about whether or not exactly n-many friends came to the party.

b. OPTION 2: The speaker is ignorant about whether or not exactly 4 or more than 4 friends came to the party.

IIs like (13a) are mentioned in Mayr (2013: 158), and those in (13b) were first argued for by Büring (2007).[1] As our first task, we should adjudicate between these two options to find the one that better characterizes IIs with SMs.

Both options characterize SMs in terms of a disjunctive statement, but diverge with respect to the particular disjuncts they take to participate in the statement. Let us assume a reduced domain of seven people. OPTION 1 equates the IIs of a sentence like (12) with a list of multiple disjuncts as in (14a). According to OPTION 2, the IIs of (12) are equivalent to a disjunction like (14b).

(14) a. OPTION 1: Either four or five or six or seven people came to the party.

b. OPTION 2: Either four or more than four people came to the party.

We can use what we know about disjunction as a benchmark to assess which of the two options in (14) better matches our intuitions regarding SMs. As is well known, disjunctions can convey an II that the speaker does not know which of the disjuncts is true. Consider:

(15) Bill read Tintin or Asterix.

Upon hearing a sentence like (15), the addressee may draw a number of conclusions. Assuming the speaker is correct, she knows that Bill read Tintin or Asterix, which by itself is consistent with the possibility that Bill read both. However, because the speaker does not specify which comic Bill read, the addressee may draw an inference that the speaker does not know which comic Bill read. This is true of simple as well as multiple disjunctions. The existence of this kind of IIs is responsible for the oddness of texts where a disjunction is followed by a statement resolving the question as to what disjunct is true.

1 Geurts & Nouwen (2007: 558) and Coppock & Brochhagen (2013b: 10) also suggest that these are the right IIs for SMs that modify elements that participate in non–entailing scales.

(16) a. Bill read Tintin or Asterix, #in fact he read Asterix.
b. Bill read Tintin, Asterix or Blueberry, #in fact he read Asterix.

Surely, either proposition in (16) could be verified in a scenario where the speaker knows exactly which comic Bill read; in this sense, both propositions are *verifiable* in such scenarios. However, neither proposition is *assertible*, in the sense that the amount of knowledge possessed by the speaker exceeds that which is required by the felicity conditions of disjunction–assuming, of course, all the pragmatic principles are in place.

Specifically, we can assume that a cooperative speaker will felicitously utter ϕ if she knows/believes that the proposition conveyed by ϕ is true, or she has good enough grounds to believe so (Hintikka 1962, Grice 1975, Gazdar 1979, a.o). Suppose further that ϕ is of the form $[\psi \vee \chi]$. In that case, what does it mean to know or to have evidence enough for $[\psi \vee \chi\ ]$? The minimal assertibility conditions for disjunction seem to be essentially modal: they require that the speaker considers that both ψ and χ are possibly (but not certainly) true (cf. Zimmermann 2000). Thus, asserting ϕ requires that ϕ is true, whereas asserting $[\psi \vee \chi]$ requires both that $[\psi \vee \chi]$ is true and that each proposition $[\psi]$ and $[\chi]$ is potentially true. Similar observations hold for multiple disjuncts: each disjunct must constitute an epistemic possibility for the speaker (Alonso-Ovalle 2006). These intuitions seem to be essentially correct, as suggested by the examples in (16) above.

With these background assumptions at hand, we can now see how the two options in (13) make different predictions about the inferences that the listener is allowed to draw after hearing an SM statement. If a disjunctive statement requires the speaker to believe in the possibility that each disjunct might be true, then OPTION 1 1 requires of the speaker a much greater epistemic commitment than OPTION 2. (Baseline sentence is (12) above.)

(17) a. OPTION 1: The speaker believes that it is possible that exactly four people came to the party, and that exactly five people came, and that exactly six people came, and that exactly seven people came.
b. OPTION 2: The speaker believes that is possible that exactly four people came to the party and she believes that it is possible that more than four people came to the party.

To put differently, the IIs conveyed by OPTION1 1 are "total" whereas the IIs conveyed by OPTION2 are "partial" (Nouwen 2015, Schwarz 2016a): the IIs conveyed by (17a) are about each disjunct in the full range of possibilities, whereas those of (17b) are not. As a consequence, only on OPTION 2 is a certain degree of knowledge compatible with an SM-statement.

2.1 The numeral case

Let us first consider SMs with numeral associates, like the example we saw above in (12). OPTION 1 predicts that when SMs combine with numerals, the speaker is ignorant with respect to every number in the domain above (for *at least*) or below (for *at most*) the one that is mentioned. Consider:

(18) **Situation:** *Two commentators are talking on TV about a classic basketball game played in the 90's. They are commenting on the points that were scored in that game on triples. A commentator says: Michael Jordan scored at least 30 points.*

Both commentators know that triples are three-point field goals in basketball, in contrast to the two points awarded for easier shots. They assume, too, that they are targeting an audience that is well versed in the rules of basketball, and so this information is shared by every agent in the conversation, active or passive. In this situation, the commentator's utterance is perfectly acceptable. This is an instance of partial ignorance: the addressee cannot draw an inference that the speaker is completely ignorant, since she knows that the speaker does know something–namely, that quantities of scores that are not tuples of three are not allowable options. This is not possible with overt disjunction. In the same context, (19) is odd:

(19) Michael Jordan scored 30, 31, 32 or 33 points.

If OPTION 1 were on the right track and SMs conveyed total ignorance, one would expect parallel behavior with (18) and (19), which in this case would amount to parallel infelicity. However, the felicity of (18) suggests that SMs convey partial ignorance–the speaker need not be ignorant about whether Michael Jordan scored *n* points for every number *n* above 30. Thus, OPTION 2 seems to be a better approximation of this property of IIs with SMs.[2]

Another manifestation of partial ignorance can be seen when an SM is used in situations where the bounds denoted by SMs are flexible. Consider the following example (from Nouwen 2015):

2 One could think of an alternative to rescue OPTION 1 by rejecting those points in the scale that stand in conflict with world knowledge or common ground information. If that were the case we would have to know why overriding IIs of SMs is harder than overriding IIs of disjunction, given that both type of IIs seem to be disjunctive at heart. One might still think that disjunctions and SMs are different, in that in multiple disjunctions the relevant alternatives that must be alive in the speaker's epistemic state are "mentioned". Although, intuitively clear what it is meant by "mentioning an alternative", it requires to be spelled out. Moreover, such option would be problematic with the data reviewed in section §2.2 below. Instead, no such explanation is necessary if we accept that SMs convey partial ignorance.

(20) **Situation:** *Bill forgot the password of his WIFI network. The only thing he remembers is that the password is between six and ten characters long.*

a. The password is at least six characters long.

b. The password is at most ten characters long.

In the situation above, Bill can utter both (20a) and (20b) felicitously, even though his epistemic state excludes some of the values that in principle could be available. Moreover, the speaker can utter both sentences without bringing attention to any misleading implicatures. In this regard, SMs differ again from multiple disjunctions, which would demand that the speaker be ignorant with respect to every number above/below six.

So far, we have reasons to believe that OPTION 2 2 is superior to OPTION 1 in characterizing the IIs of SMs when they associate with numerals. However, to be certain that OPTION 2 is on the right track, we also need to ensure that SMs are indeed infelicitous when the particular disjuncts that participate in the description of the IIs are contradicted. There are two such disjuncts for each of the SMs.

(21) a. $[\textit{at least } n] \rightsquigarrow [\textit{exactly } n] \vee [\textit{more than } n]$

b. $[\textit{at most } n] \rightsquigarrow [\textit{exactly } n] \vee [\textit{less than } n]$

The following examples suggest that the two disjuncts mentioned in (21) seem to have a preferential status, as the possibility of them being true seems to be necessarily included in the epistemic state of the speaker.

(22) Bill ate at least two apples...

a. #but I know that he didn't eat only two.

b. #in fact, he did not eat more than two.

c. but I know that he didn't eat {four/three or four/between three or six/...}.

(23) Bill ate at most six apples...

a. #but I know that he didn't eat exactly six.

b. #in fact, he did not eat less than six apples.

c. but I know that he didn't eat {four/three or four/between two or five/...}.

The examples in (22c)/(23c) are not surprising, given the fact, already established, that SMs convey only partial ignorance, and so the speaker need not consider each greater or lesser number other than the one modified by the SM to be a possibility. The examples in (22a)/(23a) show that part of what the speaker must know in order to felicitously utter a sentence with an SM is that the number that the SM associates with constitutes a possibility. Similarly, (22b) and (23b) show that there must be

some number of apples besides the number expressed by the SMs' associate such that Bill could have eaten that many apples.

Summary For SMs with numeral associates to be felicitous, the speaker need not be totally ignorant, but she must be ignorant about certain information. Specifically, we found that when uttering a sentence where an SM modifies a numeral *n*, there must be at least two possibilities that the speaker has to consider to be true: *exactly n* and *more than n* in the case of *at least n*, and *exactly n* and *fewer than n* for *at most n*. These patterns correspond to what is predicted by OPTION 2, as originally proposed by Büring (2007), and cannot be as in OPTION 1.

We can informally summarize the assertibility conditions of SMs as follows (cf. Cohen & Krifka 2014, Spychalska 2015):

(24) *Assertibility conditions of* at least *with numerals*:
A proposition ϕ of the form *at least n P* is assertible by a speaker *S iff*:

a. *S* knows that ϕ is true,

b. *exactly n P* is compatible with all *S* knows,

c. *more than n P* is compatible with all *S* knows, and

d. *S* knows that *less than n P* is not the case.

(25) *Assertibility conditions of* at most *with numerals*:
A proposition ϕ of the form *at most n P* is assertible by a speaker *S iff*:

a. *S* knows that ϕ is true,

b. *exactly n P* is compatible with all *S* knows,

c. *lessthan n P* is compatible with all *S* knows, and

d. *S* knows that *more than n P* is not the case.

These assertibility conditions correspond to what the epistemic state of a cooperative speaker has to be like so that a sentence with a SM-numeral can be uttered felicitously. (Again, this is so assuming a situation where all pragmatic principles are at work.) To this respect, the assertibility conditions of SMs are fully parallel to those of disjunction, where each disjunct is required to be possibly true, and it is required not to be certainly true by the speaker. [3]

3 What the speaker knows for certain amounts to what the SMs assert, namely the lower (for *at least*) and upper (for *at most*) bounds of the range that delimits the space of available possibilities. That this is the semantic contribution of numerals modified by SMs is part of the standard analysis in terms of Generalized Quantifiers, as first presented in Barwise & Cooper (1981): $[\![\textit{at most } n]\!] = \lambda A.\lambda B\ .\ |A \cap B| \leq n$ and $[\![\textit{at least } n]\!] = \lambda A.\lambda B\ .\ |A \cap B| \geq n$.

2.2 Ignorance beyond numerals

Our previous discussion offers us an answer to the question about PREDICTABILITY, posed in section §1, repeated below:

(6) PREDICTABILITY
Is there any proposition in particular about which the speaker must be ignorant about so that she can successfully use an SM?

With both SMs, *at least* and *at most*, the exhaustive interpretation of the prejacent to the sentence uttered must necessarily constitute an epistemic possibility for the speaker (i.e., that *exactly n* is true). For *at least*, that *more than n* must as well be a mandatory possibility, as in *less than n* for the SM *at most*. Moreover, these epistemic possibilities are pragmatically strengthened to form IIs, and so we can conclude that the answer to PREDICTABILITY in the case of numerals is positive: the speaker must be ignorant about whether *exactly n* or whether *more/less than n* is the case.

We can now aim to answer our second question, concerning UNIFORMITY.

(11) UNIFORMITY
Are the inferences that come with SMs the same across the board, regardless of the associate type of the SM?

As mentioned before, a tacit assumption in the literature is that IIs of SMs is uniform across associate types. As we saw in examples (7)–(10), SMs seemed to convey the same kind of IIs irrespective of the differences in associate types. However, we now have a more precise characterization of IIs, and are equipped to ask whether the assertibility conditions stated in (24) and (25) are the same across different associate-types.

As a first case, let us consider SMs with quantiers like some, as in (7a). The sentence conveys that the speaker is ignorant about how many students came, whether all of the students came, etc.

(7a) At least some students came to the party.

We can examine these IIs using the same tests employed above to identify the assertibility conditions of SMs with *some*. (Bear in mind that reproducing the judgments with *some* may require some more effort, most likely due to the inherent vagueness of the determiner).

(26) Bill ate at least some apples...

a. #but I know that he didn't eat just some.

b. #in fact, he didn't eat many.

c. but I know that he didn't eat {most of them/all of them}.

Suppose that what is at stake is how many of the apples Bill ate. Suppose further that you knew that Bill ate almost all the apples, that is, that he ate more than just some apples. Then you would be quite misleading in saying that he ate *at least some*, since you could have been more informative. This clash between information known and information conveyed is what is brought by the follow-up in (26a). Similarly, if you knew that Bill barely ate a handful of apples, that is, that he did not eat more than just some, by saying *at least some* you would be conveying the false possibility that he ate more than just some apples, maybe many apples, maybe even all of them. Being in such an epistemic state, however, clashes with an *at least some* utterance in the same way that numerals do (26b). Finally, (26c) shows that no conflict arises when both knowledge about *at least some* and the possibility that *more than just some* are conveyed, suggesting that IIs conveyed by SMs with some are also partial. (The reader is invited to check the facts with other quantifiers). This is exactly the same kind of behavior we observed with numerals: the exhaustive interpretation of the prejacent must constitute an epistemic possibility for the speaker, and so does the next relevant more informative alternative (i.e., in this case, many could be more than just some). Once again, these IIs are the correlate of the OPTION 2 (13) above.

We cannot yet conclude in favor of UNIFORMITY. So far, the conclusion seems to be that associates like *some* share the relevant properties of numeral associates in what they demand the speaker's epistemic states to be (with respect to II calculations). This is no coincidence: we must note that numerals and quantifiers like *some* share two important logical properties, relating to the fact that they are both ordered by logical strength, i.e. they constitute Horn scales (Horn 1972). First, for any term in $\{1,2,3,4,5,6...\}$ or {*some, many, most, all*}, instances of *x A are B* with one number/determiner term *x* semantically entail instances with any term to the left, but not to the right, as illustrated in (27).

(27) a. Bill ate five apples $\vDash$ Bill ate four apples

b. Bill ate many apples $\vDash$ Bill ate some apple

Second, Horn scales are totally ordered: for any member of a set that is totally ordered, it either entails or is entailed by every other member in the set. For instance, for any natural numbers n, i, an expression *n As are B* entails *n-i As are B* and is entailed by *n+i As are B*; moreover, for every number *j*, *j As are B* either entails or is entailed by *n As are B*.

However, SMs can associate with elements that fail to have one or both of these properties. For instance, SMs can associate with scales where, unlike with Horn scales, the relation between its members is not driven by logical entailment. These are often called "ad hoc", "pragmatic" or "lexical" scales (cf. Hirschberg 1985), as they compose of elements that "outrank" each other but are nevertheless mutually exclusive; i.e. they are non-entailing scales. Take for instance the lexical scale established by professorship ranks at US universities: *visiting professor*, *assistant professor*, *associate professor*, *full professor*, etc. One cannot be an associate professor and a full professor at the same time, and yet one cannot be a full professor without having been an associate first. In this sense there is a common understanding that full professors outrank associate professors, but these ranks are not ordered by entailment.

(28) a. Bill is an associate professor $\not\models$ Bill is an assistant professor
b. Bill won a silver medal $\not\models$ Bill won a gold medal
c. Bill is a sergeant $\not\models$ Bill is a private

These pragmatic scales also trigger IIs, and so the sentence (29) below might convey ignorance as to the exact rank Bill holds.

(29) Bill is at least an associate professor.

An inspection of non-entailing scales with respect to the assertibility conditions obtained for numerals and determiners, reveals that they have parallel assertibility conditions.[4]

(30) Bill is at least an assistant professor...
a. #but I know that he does not have tenure.
b. #in fact, he has tenure.
c. but I know that he is not {an associate/a full} professor.

The (30a)/(30b) examples are infelicitous for the same reasons that the determiners were: it seems that asserting that Bill is at least an assistant professor commits the speaker to the possibilities that Bill is in fact an assistant professor and that he might be of a higher rank. Thus, (30a) should be bad with any continuation conveying knowledge of tenure status. In turn, (30c) shows that ignorance can still be partial. As with the determiner case, no differences are found with non-entailing scales either, and so entailment cannot be a decisive factor in determining the implicative patterns of SMs'.[5]

4 In the US academic system only associate and higher ranked professors have been granted tenure.
5 By the same token, SMs modifying evaluative scales behave alike. Take, for instance, a preference scale formed by potential vacation destinations that Bill is considering, *Springfield* $\gg$ *NYC* $\gg$

SMs can also associate with elements that lack the second property, namely total ordering. As an illustration, let us consider conjunctive plurals. Assuming a domain with individuals {*a, b, c, d*}, an expression of the form *a⊕b are B* neither entails nor is entailed by *a⊕c are B*. [6] When dealing with partial orders it is useful to have names for those pairs of members that are not in an entailing relationship. For any two elements x, y of a partially ordered set P, if $x \leq y$ or $y \leq x$, then x and y are "comparable". Otherwise they are "incomparable". Thus, $a \oplus b$ and $a \oplus c$ in the previous example are incomparable, whereas $a \oplus b$ and $a \oplus b \oplus c$ are.

Let us take an example using an SM with a conjunctive plural associate, as in (8a), repeated below, which conveys ignorance as to who exactly came to the party. For present purposes, assume the reduced domain {*Bill, Sue, Liz*}.

(8a) At least Bill came to the party.

When we examine the assertibility conditions of SMs modifying conjunctions, we find a difference compared to the other cases we have seen so far. Recall that with totally ordered scales, the exhaustive interpretation of the prejacent had to obligatorily constitute an epistemic possibility for the speaker. But compare this with (32) in the context of (31) below.

(31) **Context**: Sherlock Holmes went on vacation for a couple of days and let some his friends celebrate a dinner on 221B Baker Street: Dr. Watson, Mrs. Hudson, Mycroft, Irene Adler and some of the Baker Street Irregulars. After vacation, he returns to his room only to discover that somebody has been messing with his chemistry set. Inspector Lestrade from Scotland Yard is with him, and asks:

Rockport ≫ *Jamaica*. Obviating the concessive reading (Nakanishi & Rullmann 2009, Biezma 2013), a speaker can express ignorance as to where exactly Bill went with a sentence like *Bill went at least to NYC*–for instance, in a situation where economic matters where decisive. The knowledge that is compatible with such utterance in an ordinary conversational situation parallels that of determiners and other lexical scales seen above.

(i) a. #but I know that he couldn't afford going to Rockport or Jamaica.
b. #in fact, he went to Jamaica.
c. but I know that he did not go to {Rockport/Jamaica}.

These kind of scales can be taken to be a variant of the pragmatic scales alluded to above, ones where the ordering is not provided by common knowledge but by idiosyncratic properties and attitudes of the agents participating in the conversation.

6 More explicitly, total orders are antisymmetric, transitive and total: for elements x, y, z of a totally ordered set T, if $x \leq y$ and $y \leq x$, then $x = y$; if $x \leq y$ and $y \leq z$, then $x \leq z$; and either $x \leq y$ or $y \leq x$. Partial orders are also antisymmetric and transitive, but they are reflexive instead of total, a weaker condition: all elements x in a partially ordered set P are such that they are related to themselves, $x \leq x$.

(32) IL. Who do you think that touched the chemistry set?
SH. It was at least Mycroft and Mrs. Hudson, but not only them.

There does not seem to be anything wrong with Sherlock's answer in (32). Set in an ordinary detective dialog, we can take his contribution to be maximally informative: from his answer, Inspector Lestrade can learn that Sherlock knows that Mycroft and Mrs. Hudson did touch the chemistry set, but that they were not the only ones in doing so. That is, saying that at least them both touched the set is not at odds with an epistemic state where it is taken for granted that somebody else touched it as well. This is unlike the behavior of numerals, as a comparison to a numeral version of the same dialog in the same scenario reveals.

(33) IL. How many people do you think that touched the chemistry set?
SH. #It was at least two people, but not only two.

The contrast between (32) and (33) is clear, and (33) with a numeral fails just like they did before–assuming, that is, that Sherlock is not feeling playful and he is being genuine and maximally informative, given the evidence. The critical point to be learned from this contrast is the following: when SMs associate with a partially ordered associate, the exhaustive interpretation of the prejacent can but need not be among the epistemic possibilities considered by the speaker.[7]

Of course, Sherlock's answer in (32) is perfectly adequate also in the eventuality that he happened to know for a fact that there was somebody in particular who could have not touch the set; for instance, if he knew that it could have not been Irene Adler. This is the signature of partial ignorance again (cf. OPTION 1 in (13) above). Denying all more informative alternatives to the prejacent is the one situation that makes SMs with partially ordered associates crash:

(34) #It was at least Mycroft and Mrs. Hudson, and nobody else.

Summary An examination of non-numeral associates reveals that the assertibility conditions of SMs modifying a partially ordered scale differ significantly from those of totally ordered scales. More concretely, sentences of the form *SM n P* with some number *n* are only felicitous if the speaker takes the correspoding proposition *only/exactly n P* to be compatible with all she knows. In turn, this is not a requirement

7 The intuitive contrast between the answers in (32) and (33) is supported by recent results from experimental research. Mendia (2016b) presents a study concerned precisely with the status of the exhaustive interpretation of the prejacent in SM-statements. Using a truth-value judgment task the author shows that speakers accepted sentences of the form *at least a⊕b P but not only a⊕b P* far more often than the numeral counterparts *at least n P but not only n P*. The reader is referred to that paper for further details.

when SMs modify conjunctive plurals. The assertibility conditions of SMs with conjunctions can be informally stated, then, as follows:[8]

(35) *Assertibility conditions of* at least *with conjunctive plurals*:
For a domain D $\{a, b, c, d\}$, a proposition ϕ of the form *at least* $a{\oplus}b$ P is assertible by a speaker S *iff*:

a. S knows that ϕ is true,

b. there is some x in D such that $a{\oplus}b{\oplus}x$ P is compatible with all S knows,

c. S knows that no x part of $a \oplus b$ is such that *only* x P.

(36) *Assertibility conditions of* at most *with conjunctive plurals*:
For a domain $\{a, b, c, d\}$, a proposition ϕ of the form *at most* $a{\oplus}b$ P is assertible by a speaker S *iff*:

a. S knows that ϕ is true,

b. there is some x part of $a \oplus b$ such that *only* x P is compatible with all S knows,

c. S knows that there is no x in D such that $a{\oplus}b{\oplus}x$ P is true.

2.3 Two answers

The first goal of the paper was to scrutinize the properties of the IIs that come with SMs and, on the way, to answer two questions about the nature of these IIs; we dubbed these questions PREDICTABILITY and UNIFORMITY.

(6) PREDICTABILITY
Is there any proposition in particular about which the speaker must be ignorant about so that she can successfully use an SM?

(11) UNIFORMITY
Are the inferences that come with SMs the same across the board, regardless of the associate type of the SM?

We found out that there are minimal epistemic conditions that speakers must meet to successfully use SMs; in turn, these minimal conditions constitute what is minimally predictable about the speakers epistemic state. The answer to the PREDICTABILITY question must be in the positive, then.

With respect to the UNIFORMITY question, we found out that the answer should be negative: SMs with totally ordered associates trigger different IIs as compared to

8 The expression "part of" can be understood in an intuitive way, akin to Link's (1983) two-place predicate $\prod$, the individual part relation, satisfying the biconditional $a \prod b \leftrightarrow a \oplus b = b$.

SMs with partially ordered associates. The locus of the difference lies in what is required of the exhaustive interpretation of the prejacent: the speaker **must** necessarily take this epistemic possibility into account with totally ordered scales, but she **need not** do so with partially ordered scales.

The lack of UNIFORMITY in SMs' inferences has an impact on the answer we provide for PREDICTABILITY as well, since both types of associates differ in the kind of ignorance that is predictable in each case. With totally ordered associates, listeners confronted with a SM-statement are able to immediately draw the inference that the speaker is ignorant as to whether the exhaustive interpretation of the prejacent is the case. With partially ordered associates, however, this is not the case, and all listeners are able to infer is that there is some Stronger Alternative that the speaker is ignorant about.

3 Laying the groundwork

The discussion in section §2 tells us that an adequate theory of SMs must account for two key properties. First, it must account for the fact that ignorance with SMs is partial, but with certain minimal conditions on speaker ignorance. Second, it must account for the fact that the nature of these minimal conditions differ depending on whether the associate constitutes a total or partial order. In what follows, I will present an account of SMs that derives both properties. The analysis builds on a classical neo–Gricean implementation. I will argue that the calculation of implicatures with SMs requires two sets of alternatives (Mayr 2013, Schwarz 2016a), provided by two independent mechanisms: the familiar substitution method within elements of a Horn Set (Horn 1972, Sauerland 2004b), and Association with Focus, whereby a set of alternatives is obtained by replacing the focus-bearing constituent with contextually relevant alternatives (Rooth 1985, Fox & Katzir 2011).

Before turning to the formal implementation of this analysis, however, two key background assumptions need to be justified. The first has to do with the pragmatic treatment of IIs, the motivations for which is laid out in section §3.1. The second concerns Association with Focus, which is necessary for deriving one of the two sets of alternatives needed for II-computation. I will provide a number of arguments in section §3.2 showing that SMs are conventionally associated with focus.

3.1 Ignorance is pragmatic

As indicated by their assertibility conditions in (24-25) and (35-36), there is a tight connection between the felicity conditions of SMs and the speaker's communicative intentions. In fact, it could be argued that SMs's contribution to the discourse in non-embedding contexts is primarily to convey the speaker's ignorance (Coppock &

Brochhagen 2013b). Yet, under certain circumstances, SM-statements seem felicitous even when their assertibility conditions are denied or blatantly unmet. This behavior bears the blueprint of a conversational implicature (Grice 1975, Gazdar 1979, a.o.; particularly as formulated in Hirschberg 1985), and suggests that IIs of SMs should be understood as such. This is the stand taken by a number of authors (Büring 2007, Coppock & Brochhagen 2013b, Mayr 2013, Kennedy 2015; see especially Schwarz 2016a) and the one that will be adopted here. Specifically, I will take IIs with SMs as a product of the listener's reasoning about alternative utterances that the speaker chose not to produce, in a neo-Gricean framework.

A key property of such implicatures is that very often they are optional, and if IIs fall into that category, we should expect optionality there, as well.[9] A quick application of the basic tests seems to support this view: IIs with SMs are both cancelable and reinforceable.[10]

(37) CANCELABILITY:
Context: Bill has four kids. Yesterday he saw a sign at a supermarket: "Huge sales and discounts for parents. To qualify, you must have at least three kids." After reading it, Bill reasoned as follows: "I qualify, I have at least three kids. In fact, I have four."

(38) REINFORCEABILITY:
Bill has at least three kids, but I have no idea how many exactly.

Other arguments that IIs must be pragmatic are a little more involved and rely on cooperative notions that are at the heart of the Gricean enterprise. As Grice (1975) pointed out–referring to disjunctive sentences–IIs may be canceled when it is known by all the participants in the conversation that the speaker is not being maximally informative. These are cases where the speaker is not expected to provide all the relevant information that is available to her, in whichever form. The reason is the following: usually, it is taken to be shared knowledge that participants in a conversation obey some version of the Maxim of Quantity. The Maxim of Quantity

9 It is understood that optionality in general, and cancellability in particular, is not a sufficient condition to identify conversational implicatures. Sadock (1978), for instance, points out one of the meanings in ambiguous sentence can often be canceled, as in *everyone speaks only one language although no one language is spoken by everyone*. And presuppositions as well seem to be cancellable (von Fintel 2008). So, with Hirschberg (1985), one should be cautious and take optionality as a necessary requirement of conversational implicatures.

10 In this respect, SMs behave like disjunction: in some cases disjunction does not trigger an II, even in plain non-modal contexts. The following example is from Chierchia (2013), adapted from von Fintel & Gillies (2010): *We lost a ball. John is telling us that it is not in Box A. We saw it land in Box A or Box B; thus, the ball must be in Box B.* It seems that in the context above the inference that the speaker does not know which of the disjuncts is true has to be defeasible to make a coherent discourse.

is a cooperative principle stating, roughly, that the speaker is expected to convey all the information she has available, that is, she is expected to provide the strongest relevant statement she is able to. As Horn (1984: 13) put it, *say as much as you can without saying more than you must*. For concreteness, assume Grice's (1975) own formulation.

(39) MAXIM OF QUANTITY

1. Make your contribution as informative as required.
2. Do not make your contribution more informative than is required.

IIs arise as a direct consequence of the mutual agreement that the Maxim of Quantity is active. Therefore, in the absence of the assumption that the speaker is following the Maxim of Quantity, speakers are not expected to be maximally informative; there could be stronger relevant propositions that they could have remained silent about while still being cooperative. As it turns out, in situations where it is known that the speaker is not obeying the Maxim of Quantity, IIs are not present. Grice (1989: 44) discusses the case of disjunction:

> 'I can say to my children at some stage in a treasure hunt, "The prize is either in the garden or in the attic. I know that because I know where I put it, but I'm not going to tell you." Or I could just say (in the same situation) "The prize is either in the garden or in the attic", and the situation would be sufficient to apprise the children the fact that my reason for accepting the disjunction is that I know a particular disjunct to be true.'

The treasure hunt scenario illustrates that the cancelation of the II is contingent upon knowing whether the different agents in the conversation have agreed on obeying the Maxim of Quantity or not. This is of course a property of the conditions under which the discourse unfolds, and SMs are no different to this respect. IIs of SMs have the property of disappearing under certain circumstances, suggesting a heavily context dependent nature. As an illustration, consider the game show scenario (adapted from Fox 2014):

(40) **Situation:** In a TV game show, utterances by the host are presupposed to disobey the Maxim of Quantity. The contestant has won the biggest prize, which consists of one of two options: She either takes $5000 in cash or she takes an envelope with an amount of cash unknown to her, but that the audience and the host already know. The contestant has to gamble. At some point, the host decides to give a hint that will help the contestant to assess her chances of picking the most profitable choice. Of course, the hint is such that it only provides part of the information available to the host, and this is

common understanding for both the contestant and the audience. In this case, the host says: "The envelope contains at least $2500."

What is important about the host's utterance is that in no way is he trying to convey her ignorance about the situation, nor is she trying to be misleading. Deactivating the Maxim of Quantity thus makes the hint provided by the host appropriate even though she knows the exact quantity that the envelope contains. And, more importantly, it precludes the contestant from drawing the inference that the host does not know how much money it contains. In fact, the contestant can be confident that the hint is true precisely because she takes the host to be an authority on the matter. Thus, just like in the treasure hunt scenario, IIs of SMs depend on the assumption that the speaker is being maximally informative; when discourse participants drop this assumption, no IIs arise.

Knowing whether or not the speaker is being maximally informative is not part of one's linguistic competence, as one might gain such knowledge from several sources, ranging from world knowledge to tacit one-time assumptions between agents. To put differently, there is simply no way to tell from an SM utterance alone whether the speaker is being maximally informative with respect to her epistemic state. The fact that IIs are crucially dependent on the goals of the conversation and on assumptions about the speakers intentions gives further support to the idea that these inferences are pragmatic in nature.

3.2 The role of focus

A second key ingredient in my analysis of SMs is the idea that focus serves a mediating role between their semantics and the IIs they give rise to. This link, I will argue, can shed light on when IIs are present/absent and also the precise nature of the IIs conveyed.

3.2.1 A tight connection

I start off with two independent observations. First, notice that the inferences that the addressee is allowed to infer are restricted by focus: IIs must covary with the associate of the SM, which in turn is determined by focus marking (Krifka 1999). This is easier to see when SMs can associate at a distance with different focused elements in otherwise identical sentences (examples from Coppock & Brochhagen 2013a).

(41) a. The chair { at least / at most } invited the postdoc$_F$ to lunch.
⇝ ignorance about whether {someone else/someone} was invited

b. The chair { at least / at most } invited the postdoc to lunch$_F$.
⇝ ignorance about whether she got invited to {something else/anything}

This is a strong requirement: (41a) cannot carry the IIs of (41b) or vice versa.

The second observation is that IIs disappear when no focus is predicted under the scope of the SM. For instance in the short dialog in (42), the sentence (42b) directly answers the question in (42a): focus in (42b) as an answer to (42a) is predicted to be on [ten students] (see e.g., Rooth 1992, Roberts 2012), which in turn correlates with what the speaker is ignorant about, in this case the exact number of students that took Linguistics 101 (examples from Westera & Brasoveanu 2014).

(42) a. Exactly how many students took Linguistics 101?

b. {At least/At most } ten students took Linguistics 101. [=(43b)]

In contrast, in (43b) the associate of the SM does not directly answer (43a) and no constituent is predicted to bear focus. Correspondingly, the listener need not derive any II.

(43) a. Did {at least/at most } ten students take Linguistics 101?

b. {At least/At most } ten students took Linguistics 101. [=(42b)]

In sum, SMs seem to associate with focused elements and their IIs covary with the constituent that bears focus. In addition, when the associate of an SM does not bear focus, IIs need not arise. These two observations suggest that focus plays a pivotal role in determining what the IIs are about, and when they are present. I will argue that this intimate link is best captured by analyzing SMs as conventionally Associated with Focus. Below, I offer several pieces of empirical evidence supporting such a view. The reader is referred to Mendia (2016a) for further evidence that a weaker relationship with focus (as in the case of *-est*, the bare form of the superlatives) is insufficient.

3.2.2 Evidence for conventional meaning

The diagnostics in this section are drawn from Beaver & Clark (2008) and others to support the claim that SMs are Conventionally Associated with Focus (C-AwF), whereas the bare superlative form is Freely Associated with Focus (F-AwF). This difference in the degree of focus association that each expression bears parallels that of some focusing adverbs (e.g., *only* and *always*). Thus, to make the discussion clearer, I will draw parallels between SMs and the C-AwF element *only*, and between

bare superlatives and the F-AwF *always*, and then show that both pairs of expressions behave differently from each other.[11]

Association with weak elements As Beaver & Clark (2008) pointed out, C-AwF expressions like *only*, *even* and *also*, are unable to associate with prosodically weak elements, like reduced pronouns. F-AwF like *-est* or *always* are able to associate with weak forms. The association patterns in (44) suggest that SMs pattern with C-AwF elements like *only*.

(44) **Context**: You can see Mrs. Hudson, but do you see Sherlock and Watson?

a. Well, I {**always/most often/least often**} { see'em / see them}.

b. I can {**only/at least/at most**} { *see'em / see them}.

A consequence of this ban is that some meanings cannot be obtained when coventionalized elements try to associate with a weak form. In the examples below, the sentence with *only*, but not those with *always* or *-est*, are odd–the context is set such that the intended interpretation requires association with a reduced form, which is blocked.

(45) **Context**: You discussed a lot with Sandy. Of all the times you talked with her, how often were Fred and Sue the people you talked about?

a. I **always** talk about'em
   ⇝ *whenever I discussed someone with Sandy, I discussed Fred and Sue.*

b. I talk about'em **the least**
   ⇝ *I discussed Fred and Sue less often than anybody else.*

c. I talk about'em **the most**
   ⇝ *I discussed Fred and Sue more often than anybody else.*

d. #I **only** talk about'em
   ↛ *I only discussed Fred and Sue (and no one else) with Sandy.*

The same effect that *only* has above can be reproduced with SMs. The paraphrases below each sentence provide the targeted meaning.

11 It should be noted that some of the grammaticality judgments that I present here are subject to speaker variation. I have always tried to provide minimal pairs with conventional *vs.* free association, such that we can reason from a contrast, rather than categorical judgments of acceptability.

(46) **Context**: Fred has discussions with Jane very often. Bill and Sue are their officemates, so it's likely that they talk about them. Who else do you think they talk about?

a. Well, I'm not sure, but I know that they **at least** talk about them.
⇝They talk about Bill and Sue (and maybe somebody else).

b. ?Well, I'm not sure, but I think that they **at most** talk about them.
⇝If they talked about anyone, they talked about Fred and Sue, and no one else.

c. #Well, I'm not sure, but I know that they **at least** talk about'em.
⇝They talk about Bill and Sue (and maybe somebody else).

d. #Well, I'm not sure, but I think that they **at most** talk about'em.
⇝If they talked about anyone, they talked about Fred and Sue, and no one else.

Thus, SMs pattern like *only* but unlike *-est* with respect to the prosodic restrictions on the kind of elements they can associate with.

Backwards association Only a subset of the focus sensitive expressions can associate with a phrase that they do not c-command. This is usually referred to as "backwards" association. Typically, F-AwF can happen backwards, whereas only a subset of the C-AwF expressions can associate backwards (Jackendoff 1972). The examples below show that *only* belongs to the more restrictive kind, which contrasts with the more permissive conventionalized element *even*, and F-AwF elements like *always*.

(47) a. Bill$_F$ **always** reads this book.
⇝Whenever somebody reads this book, Bill reads it.

b. Bill$_F$ will **even** read this book.
⇝Even Bill will read this book.

c. Bill$_F$ will **only** read this book.
⇝ Only Bill, and nobody else, will read this book.

SMs also belong to the more restrictive kind of focus operators.

(48) a. Bill$_F$ will {**at least/at most**} read this book.
↛ at least/most Bill will read this book.

b. Bill$_F$ bought **the largest cake** for Jane.
⇝ Bill bought the largest cake for Jane than anybody else. did

Extraction Beaver & Clark (2008) noted that only F-AwF can happen with extracted elements that are realized in higher clauses. The following examples show that this is indeed the case: *always* can associate with a topicalized and a relativized element, whereas *only* cannot.[12] Once again, the same contrast can be reproduced between the bare form of the superlative and SMs: only the latter fail to associate with extracted material, and so they pattern with C-AwF expressions. The examples below show that SMs behave like C-AwF elements in their lack of ability to associate with extracted material in a number of constructions.

(49) a. Fish, I believe Kim **always** buys. F-AwF
⇝ I believe that whenever Kim buys something, he buys fish.

b. Fish, I believe Kim **only** buys. C-AwF
↛ I believe that Kim buys fish and nothing else.

(50) a. For Jane, I believe Kim bought **the biggest cake**. F-AwF
⇝ I believe that Kim bought the biggest of all the cakes for Jane.

b. Fish and wine, I believe Kim **at least** bought. C-AwF
↛ I believe that Kim bought at least fish and wine, and maybe something else.

c. ?Fish and wine, I believe Kim **at most** bought and cooked. C-AwF
↛ I believe that Kim bought and cooked at most fish and wine, and surely nothing else.

Ellipsis In English, the elision of a VP containing the associate of a C-AwF element results in ungrammaticality. This behavior contrasts with F-AwF, which can happen with elided (or implicit) material. (Example from Beaver & Clark 2008.)

(51) **Context**: At the ceremony, some soldiers salute, others fire a round in the air, some do both and others do nothing. What do Kim and Sandy do?

a. Kim **always** salutes$_F$ because Sandy always does.
⇝ Kim salutes at every ceremony because Sandy salutes at every ceremony.

12 This is true of a much wider family of constructions than what I can show here (e.g., relativization, adverb preposing, questions, inverted pseudo-clefts, among others).

b. #Kim **only** salutes$_F$ because Sandy **only** does.
↛ *Kim salutes and does nothing else at every ceremony because Sandy only ever salutes*.

Like *only*, SMs also show this restriction.

(52) **Context**: At the ceremony, some soldiers salute, others fire a round in the air, some do both and others do nothing. What do Kim and Sandy do?

a. *Kim **at least** salutes$_F$ because Sandy **at least** does.
↛ *Kim salutes and maybe fires at every ceremony because Sandy salutes and maybe fires at every ceremony*.

b. *Kim **at most** salutes$_F$ because Sandy **at most** does.
↛ *Kim salutes or does nothing because Sandy salutes or does nothing at every ceremony*.

Summary Above, I presented several pieces of evidence demonstrating that SMs show the signatures of C-AwF, in the sense of Beaver & Clark (2008). Taking association with focus to be an intrinsic part of the meaning of these expressions has a number of welcome consequences for how we derive their IIs (see Krifka 1999). Conventional Focus Association allows us to guarantee the presence of focus alternatives, which in turn can be consistently factored in when computing implicatures, allowing us to capture the observed tight connection between IIs of SMs and focus placement. More importantly, the computation of focus alternatives will allow the system to associate scales with sub-sentential expressions, a point emphasized by Geurts & Nouwen (2007) (see Coppock & Brochhagen 2013b as well).

3.3 Interim summary

In this section, I motivated the two key ingredients for my analysis of SMs: (i) that they are conversational implicatures and therefore they should be amenable to a pragmatic analysis, and (ii) that SMs show a lexically encoded conventionalized association with focus. Arguments for (i) come from the fact that IIs are optional and their presence is directly impacted by whether or not the speaker is known to be maximally informative. The claim in (ii) was motivated by the fact that (a) IIs with SMs are crucially dependent on the presence of Focus, and (b) focus placement determines the constituent in the sentence about which IIs can be drawn. Furthermore, SMs' dependency on focus was empirically supported by the fact that they show clear parallels with uncontroversially C-AwF elements like *only*.

With these ingredients at hand, we can now turn to a formal account of SMs.

4 Analysis

This section provides a unified account of the SMs *at least* and *at most* as scalar modifiers, following the idea that IIs of SMs arise as neo-Gricean conversational implicatures. Crucially, the analysis requires computing these implicatures by factoring in two sets of alternatives (Mayr 2013, Schwarz 2016a). The main innovation of the analysis presented here is that the set of alternatives relevant for the Gricean calculus is provided by two independent mechanisms. In addition to the substitution method within elements of a Horn scale (Horn 1972, Sauerland 2004b, a.o.), a different set of alternatives is obtained by replacing the focus-bearing constituent with contextually relevant alternatives (Rooth 1992, cf. Fox & Katzir 2011). In addition, I propose that Superlative Modifiers form a Horn scale with *only* (Schwarz 2016a).

4.1 Background

To properly talk about IIs, we first need to know what it means to be ignorant about something. Assume that K and P stand for the familiar epistemic certainty and possibility operators, such that $\mathsf{K}_S\phi$ means *the speaker S knows that* ϕ and $\mathsf{P}_S\phi$ means that ϕ *is compatible with all S knows*. According to the properties that Hintikka (1962) ascribed to them, both operators K and P are interdefinable, since $\mathsf{K}\phi \leftrightarrow \neg\mathsf{P}\neg\phi$ and $\mathsf{P}\phi \leftrightarrow \neg\mathsf{K}\neg\phi$.[13] In this system, the following equivalences follow: $\mathsf{K}\neg\phi \leftrightarrow \neg\mathsf{P}\phi$ and $\neg\mathsf{K}\phi \leftrightarrow \mathsf{P}\neg\phi$. Then, to be ignorant about a proposition ϕ is expressed as follows:[14]

(53) SIGNATURE OF IGNORANCE: $\neg\mathsf{K}[\phi] \wedge \neg\mathsf{K}\neg[\phi] \quad \leftrightarrow \quad \mathsf{P}[\phi] \wedge \mathsf{P}\neg[\phi]$

(53) shows the technical notion of *ignorance* that I shall rely on. To be ignorant about ϕ is a stronger notion than the mere lack of knowledge about ϕ. By *being ignorant about* ϕ I refer to a mental (epistemic) state of some agent in which she is unsure about the truth of ϕ. Crucially, in order to be ignorant about ϕ it is necessary that the agent consider both ϕ and $\neg\phi$ live possibilities compatible with her knowledge. It follows that not only does the agent not know the truth of ϕ, she also does not know the truth of $\neg\phi$. Hintikka (1962: 12–15) illustrates this difference by alluding to the contrast between *knowing that* ϕ and *knowing whether* ϕ:

13 For concreteness, I am assuming Hintikka's (1962) system, an epistemic logic developed by enriching propositional calculus with the operator K and the three additional axioms *K* (distributivity; $\mathsf{K}(p \rightarrow q) \rightarrow (\mathsf{K}p \rightarrow \mathsf{K}q)$), *T* (reflexivity; $\mathsf{K}p \rightarrow p$) and *4* (positive introspection; $\mathsf{K}p \rightarrow \mathsf{K}\mathsf{K}p$). This is a *KT4* modal system. Whether *KT4* is the most adequate logic to model knowledge and belief is a matter subject to debate among philosophers; see Hendricks & Symons (2014) for discussion.

14 I use square brackets '[]' to enclose propositions, so that if ϕ is a propositional variable, $[\phi]$ stands for *the proposition that* ϕ. In addition, I sometimes use $\mathsf{P}[\phi]$ and $\neg\mathsf{K}\neg[\phi]$ interchangeably, as well as $\neg\mathsf{K}[\phi]$ and $\mathsf{P}\neg[\phi]$, the choice depending on what expression is more intuitive on a case to case basis.

(54) a. The speaker S does not know that ϕ: $\neg \mathsf{K}_S \phi$

b. The speaker S does not know whether ϕ: $\neg \mathsf{K}_S \neg \phi \wedge \neg \mathsf{K}_S \phi$

The distinction between (54a) and (54b) is in accordance with the intuition that when we are ignorant about whether ϕ, we consider both ϕ and $\neg\phi$ to be epistemic possibilities; I take this for granted here. Sometimes I use the following notational convention, where $\mathsf{I}_S[\phi]$ means that *the speaker is ignorant about whether* ϕ:

(55) $\mathsf{I}_S[\phi] \equiv \neg \mathsf{K}_S[\phi] \wedge \neg \mathsf{K}_S \neg [\phi] \leftrightarrow \mathsf{P}_S[\phi] \wedge \mathsf{P}_S \neg [\phi]$

We turn now to the question of how to derive IIs of this form. Gazdar (1979), putting together insights from both Hintikka's (1962) epistemic logic and Grice's (1975) theory of language use, argued that IIs can be derived as clausal quantity implicatures. Assume that we are dealing with a cooperative speaker and that some version of the Maxims of Quality are at work (Grice 1975).

(56) MAXIMS OF QUALITY

a. Do not say what you believe to be false.

b. Do not say what you do not have evidence for.

The Maxims of Quality can be related to the operators K and P by Hintikka's (1962) principle of EPISTEMIC IMPLICATION, whereby utterance of a sentence ϕ by a speaker S commits S to the knowledge of ϕ: ϕ implicates ψ if $\mathsf{K}[\phi \wedge \neg\psi]$ is inconsistent.[15] When a cooperative speaker S is following the Maxims of Quality, the addressee is allowed to infer that the utterance of ϕ by S implicates that $\mathsf{K}_S\phi$. This inference is sometimes also referred to as a Quality Implicature. In order to derive IIs, however, we need some notion of logical strength. Assume then a more precise characterization of the Maxim of Quantity, defined in terms of asymmetric entailment.

(57) MAXIM OF QUANTITY
If two propositions $[\phi]$ and $[\psi]$ are such that (i) the denotation of $[\phi]$ asymmetrically entails $[\psi]$ (i.e., $[\phi] \rightarrow [\psi] \wedge \neg([\psi] \rightarrow [\phi])$), (ii) $[\phi]$ and $[\psi]$ are relevant, and (iii) the speaker believes both $[\phi]$ and $[\psi]$ to be true, the speaker should choose $[\phi]$ over $[\psi]$.

The Maxim of Quantity ensures that, given a number of true and relevant alternatives to the proposition that has been uttered, if a speaker is being cooperative, she should choose the semantically strongest alternative over the rest. In view of this definition of the Maxim of Quantity, it is useful to define the notion of Stronger Alternative

15 This notion of "implication" is closer to that of "entailment" and it is not the one that I will use when talking about implications in general.

(*SA*): An *SA* ψ of a proposition ϕ is an alternative proposition that asymmetrically entails ϕ: ψ is an *SA* of ϕ *iff* $\psi \rightarrow \phi$ and $\phi \nrightarrow \psi$. The set of *SA*s of a proposition ϕ is expressed as $SA(\phi)$ (as opposed to the set $Alt(\phi)$ of all alternatives to ϕ). According to the Maxim of Quantity, if we are to be cooperative, we have to provide the semantically strongest relevant and true proposition we can. Following the terminology in Sauerland (2004b), we now define the weakest form of inference, a Primary Implicature. In addition, we also define the Implicature Base, the set of propositions resulting from conjoining the Quality Implicature with its Primary Implicatures.

(58) PRIMARY IMPLICATURE:
The inference that $\neg \mathsf{K} \psi$, for an *SA* ψ.

(59) IMPLICATURE BASE:
The set consisting of the Quality Implicature together with its Primary Implicatures.

As an illustration of how to derive an II in this framework, consider (15) again: *Bill read Tintin or Asterix*. The sentence conveys the II that the speaker does not know which of the comics Bill read. The reasoning proceeds as follows: First, assume that the speaker is being cooperative. This means that she is observing the Maxims of Quality. Assume moreover that there is no reason to believe that the speaker is not maximally informative, and so she observes the Maxim of Quantity as well. Upon hearing (15) (represented as $[\mathtt{T} \vee \mathtt{A}]$), the addressee can conclude that the speaker thinks that this much is true. Thus, by the principle of EPISTEMIC IMPLICATION, she concludes that $\mathsf{K}_S[\mathtt{T} \vee \mathtt{A}]$. The proposition $[\mathtt{T} \vee \mathtt{A}]$ has at least two stronger alternatives, the individual disjuncts $[\mathtt{T}]$ and $[\mathtt{A}]$. This follows from the Maxim of Quantity: $[\mathtt{T}] \in SA([\mathtt{T} \vee \mathtt{A}])$, since $[\mathtt{T}]$ is relevant and $[\mathtt{T}] \rightarrow [\mathtt{T} \vee \mathtt{A}]$, but $[\mathtt{T} \vee \mathtt{A}] \nrightarrow [\mathtt{T}]$. The same reasoning applies also to $[\mathtt{A}]$. Following the Maxim of Quantity, the addressee concludes that if the speaker did not utter any one of the *SA*s, it must be because she did not have evidence enough, or maybe she did not know. Therefore, she infers the Primary Implicatures that $\neg \mathsf{K}_S[\mathtt{T}]$ and $\neg \mathsf{K}_S[\mathtt{A}]$. (60) below summarizes the process:

(60) a. ASSERTION: [(15)] = $[\mathtt{T} \vee \mathtt{A}]$
b. EPISTEMIC IMPLICATION: $\mathsf{K}_S[\mathtt{T} \vee \mathtt{A}]$
c. $SA([\mathtt{T} \vee \mathtt{A}]) = \{[\mathtt{T}], [\mathtt{A}]\}$
d. PRIMARY IMPLICATURES: $\neg \mathsf{K}_S[\mathtt{T}] \wedge \neg \mathsf{K}_S[\mathtt{A}]$
e. IMPLICATURE BASE: $\mathsf{K}_S[\mathtt{T} \vee \mathtt{A}] \wedge \neg \mathsf{K}_S[\mathtt{T}] \wedge \neg \mathsf{K}_S[\mathtt{A}]$

The Implicature Base contains all the information that the addressee may be able to deduce from the speakers utterance without any further assumptions. In particular,

according to (60e), the addressee can conclude that the speaker knows that Bill read either Tintin or Asterix, that it is not the case that she knows that Bill read Tintin and it is not the case that she knows that Bill read Asterix. These are not quite yet the IIs we want. The last step to derive the right IIs from (60e) involves deriving that each disjunct is an epistemic possibility by the speaker, i.e., $\mathsf{P}_S[\mathtt{T}]$ and $\mathsf{P}_S[\mathtt{A}]$. Luckily, the task is trivial: given the properties of the operators K and P defined above, $\mathsf{P}_S[\mathtt{T}]$ and $\mathsf{P}_S[\mathtt{A}]$ are in fact entailed by the Implicature Base.[16]

(61) ENTAILMENTS OF (60e): $\mathsf{K}_S[\mathtt{T}\vee\mathtt{A}]\wedge\neg\mathsf{K}_S[\mathtt{T}]\wedge\neg\mathsf{K}_S[\mathtt{A}]\wedge\neg\mathsf{K}_S\neg[\mathtt{T}]\wedge\neg\mathsf{K}_S\neg[\mathtt{A}]$

Thus, the Implicature Base alone provides all the necessary pieces to derive that the epistemic possibility of every disjunct is a must.[17] It follows, too, that knowledge about the truth of any one of the particular disjuncts should not be allowed, as we saw above in (16a). In this case, this happens because both $\mathsf{K}_S[\mathtt{T}]$ and $\mathsf{K}_S\neg[\mathtt{T}]$ contradict the II that $\mathsf{I}_S[\mathtt{T}]$–similarly for $\mathsf{K}_S[\mathtt{A}]$ and $\mathsf{K}_S\neg[\mathtt{A}]$.

The upshot of this discussion is that the choice of what counts as an *SA* is important: given the right choice of *SA*s, IIs may be entailed by the Implicature Base.[18] IIs of disjunctive statements can be derived by relying on independently needed formal principles, which provide the two necessary–and sufficient–ingredients to derive IIs about each particular disjunct: a suitable epistemic logic and the assumption that *SA*s are established by asymmetric entailment relations. In the following sections I extend the same pragmatic calculus to SMs.

16 To see that $\mathsf{K}[\mathtt{T}\vee\mathtt{A}]\wedge\neg\mathsf{K}[\mathtt{T}]\wedge\neg\mathsf{K}[\mathtt{A}]\rightarrow\mathsf{P}[\mathtt{T}]\wedge\mathsf{P}[\mathtt{A}]$ is the case, assume that $\neg\mathsf{P}[\mathtt{T}]$: since $\mathsf{P}[\mathtt{T}]$ is equivalent to $\neg\mathsf{K}\neg[\mathtt{T}]$, then $\neg\mathsf{P}[\mathtt{T}]$ is equivalent to $\neg\neg\mathsf{K}\neg[\mathtt{T}]$, which can be reduced to $\mathsf{K}\neg[\mathtt{T}]$ by double negative. But $\mathsf{K}\neg[\mathtt{T}]$ cannot be, since it contradicts the Primary Implicature in the premise. Thus, it must be the case that $\neg\mathsf{K}\neg[\mathtt{T}]$, which is equivalent to $\mathsf{P}[\mathtt{T}]$. (The same proof holds *mutatis mutandi* for $\mathsf{P}[\mathtt{A}]$.)

17 Above I ignored the scalar *SA* $[\mathtt{T}\wedge\mathtt{A}]$. Notice that after adding the corresponding Primary Implicature $\neg\mathsf{K}_S[\mathtt{T}\wedge\mathtt{A}]$, the Implicature Base in (60e) does not entail that $\mathsf{P}_S[\mathtt{T}\wedge\mathtt{A}]$, and so no II can be derived about $[\mathtt{T}\wedge\mathtt{A}]$. This may not be a bad thing, since implicatures associated with the conjunctive alternative to disjunctive statements can sometimes be strengthened to $\mathsf{K}_S\neg[\mathtt{T}\wedge\mathtt{A}]$ and so constitute a Secondary Implicature (or Scalar Implicature). In a classical neo-Gricean set-up, this strengthening cannot happen without further ado, usually in the form of an additional assumption ofter referred to as the "epistemic step" or "competence assumption" (see e.g., Geurts 2010). This is not to say that (15) is incompatible with the speaker's ignorance as to whether Bill read both comics.

18 This is especially relevant when we consider disjunctions with multiple disjuncts. As Alonso-Ovalle (2006) showed, in order to calculate IIs with multiple disjuncts "sub-domain" alternatives–formed by smaller disjunctions each of whose individual disjuncts are part of the assertion–must be included (see also Chierchia 2013). Once the access to sub-domain alternatives is granted, the system presented in this paper can derive IIs of multiple disjuncts just the same.

4.2 Calculating Ignorance

The previous section has set what the formal principles are that are responsible for calculating IIs. In what follows, I show how considering both Horn scale alternatives and focus alternatives provides the right kind of input to the implicature calculating mechanism so that it derives IIs of the right form in each case.

4.2.1 Step 1: Focus alternatives and truth-conditions

In order to calculate implicatures in a neo-Gricean framework, alternative propositions have to be ordered. In the case of SMs, one such ordering can be provided by asymmetric entailment relations among propositional focus alternatives. The semantics of focus delivers an ordinary semantic value and a focus semantic value that consists of a set of alternative propositions (Rooth 1985 *et seq.*). Then we can use this set of propositions to reason about plausible and more informative alternatives that the speaker could have uttered–just like we usually do in routine neo-Gricean pragmatics. I suggest that this constitutes the first set of relevant alternative propositions that is factored into the pragmatic calculus.

I take it that the conclusions obtained from section §3.2 support the claim that SMs are members of a limited class of focusing adverbs that bear a lexically determined dependency on focus, those which Beaver & Clark (2008) refer to as showing Conventional Association with Focus. (The theory of focus I am assuming correponds then to "intermediate" theory, in the sense of Rooth 1992.) Thus, SMs always make reference to focus-evoked alternatives compositionally derived throughout the semantic computation.

Informally, the meaning of a sentence S with some focalized constituent F is the set of propositions that obtains from S by making a substitution in the position corresponding to F. Alternatives to F are projected in a fully compositional fashion, hence it is type-driven mechanism (Rooth 1985).[19] This is a two tier semantic system delivering an ordinary semantic value $[\![\cdot]\!]^o$ and a focus semantic value $[\![\cdot]\!]^f$.

19 A purely type-driven semantics of focus is know to overgenerate; see Katzir (2007) and Fox & Katzir (2011) for discussion and a proposal to fix it. For the purposes of this paper, I simply assume a semantic mode of composition whereby complex expressions like $[\alpha, \beta]$ are compositionally derived by applying a function h pointwise, that is, to the meaning of $[\alpha]$ and $[\beta]$ separately (cf. Rooth 1996: 281): $[\![\alpha, \beta]\!]^f \equiv \{h(X,Y) | X \in [\![\alpha]\!]^f, Y \in [\![\beta]\!]^f\}$.

(62) a. $[\![[Sue\ ate\ [broccoli]_F]\!]^o = ate(Sue, broccoli)$
b. $[\![[Sue\ ate\ [broccoli]_F]\!]^f = \{x \in D_e \mid ate(Sue, x)\}$

As for the lexical entries of SMs, assume a propositional version whereby SMs can directly take sets of propositions as arguments (Büring 2007).[20]

(63) a. $[\![at\ least]\!] = \lambda C_{\langle st,t\rangle}.\lambda p_{\langle st\rangle}.\lambda w : \exists q[q \in C \wedge p \leq q \wedge q(w)]$
b. $[\![at\ most]\!] = \lambda C_{\langle st,t\rangle}.\lambda p_{\langle st\rangle}.\lambda w : \forall q[q \in C \wedge q(w) \rightarrow q \leq p]$

The association of SMs with focus is no longer optional, as the set C is always constrained by the focus value of their associate. In addition to an ordinary semantic value, these definitions will produce a set of propositions determined by the focus semantic value. The lexical entry in (63a) renders true a proposition p containing the SM *at least* if there is some proposition q in the relevant set of alternative propositions which is at least as strong as p and q is true in the evaluation world. In turn, a proposition p containing the SM *at most* is true if for every true proposition q in the set of alternative propositions, q is at most as strong as the prejacent p in the evaluation world.

In order to address the question of what focus sensitive expressions have in common, Rooth (1992, 1996) factors in the role of context on the semantic computation of sentences with focused constituents. In order for focus to be felicitous, the set of alternatives generated must be related to a contextually available set of alternatives C, where C is determined by contextually available or pragmatic information.

(64) Where φ is a syntactic phrase and C is a syntactically covert semantic variable, $\varphi \sim C$ introduces the presupposition that C is a subset of $[\![\varphi]\!]^f$ containing $[\![\varphi]\!]^o$ and at least one other element. [Rooth 1996: 285]

In Roothian semantics, if XP is focused its focus-semantic value is the set of all the entities of its semantic type. The squiggle focus operator effectively limits a focus semantic value $[\![\text{XP}]\!]^f$ to a contextually relevant set of alternatives C containing, minimally, $[\![\text{XP}]\!]^o$ and one other element. Consider for instance a sentence like (65a) and its corresponding LF. (For the most part I rely on the SM *at least* to illustrate the process and I turn to *at most* in section §5.3.1.)

(65) a. Sue is SM [an assistant professor]$_F$
b. LF: [$_{S^1}$ SM C [$_{S^2}$ [$_{S^3}$ Sue is [an assistant professor]$_F$] $\sim C$]]

20 These lexical entries leave a number of questions open about how to better connect the syntactic properties of SMs with their ability to semantically associate at a distance. In particular, they require displacement of the SM to a sentence initial position, and so they are insensitive to a number of limitations that SMs show (like, for instance, the inability to associate at a distance across subjects: *at least Bill ate an apple*, cannot mean the same as *Bill ate at least an apple*). Since the focus is on the IIs of SMs, I will not address these issues here. (See also footnote 22.)

Uttered in an academic context, we can assume that C as determined by $[\![$[associate professor]$_F]\!]^f$ amounts to {*assistant professor, associate professor, full professor,*...}. The derivation of the ordinary and focus semantic values proceed routinely. (I use $[\![\text{XP}]\!]^{f_c}$ to refer to the subset C of $[\![\text{XP}]\!]^f$.)

(66) a. $[\![65a]\!]^f = \{P(Sue) \mid P \in D_{et}\}$

b. $[\![65a]\!]^o =$ *Sue is an assistant professor*

c. $[\![65a]\!]^{f_c} = \left\{ \begin{array}{l} \ldots \\ \textit{Sue is a visiting professor}, \\ \textit{Sue is an assistant professor}, \\ \textit{Sue is an associate professor}, \\ \ldots \end{array} \right\}$

SMs can now apply to the meanings in (66). Consider first the case of *at least*, including explicit references to the ordinary and focus semantic values, for clarity.

(67) *Asserted content of propositions with* at least
$[\![(65a)_{\text{AL}}]\!] = \lambda w : \exists q[q \in [\![(65a)]\!]^{f_c} \wedge [\![(65a)]\!]^o \leq q \wedge q(w)]$

(68) *Asserted content of propositions with* at most
$[\![(65a)_{\text{AM}}]\!] = \lambda w : \forall q[q \in [\![(65a)]\!]^{f_c} \wedge q(w) \rightarrow q \leq [\![(65a)]\!]^o]$

Minimally, both definitions convey a semantic bound on the range of values allowable in each case. Thus, in a context where Bill has three dogs, the definitions correctly capture that the following sentences should be false.

(69) a. Bill has at least four dogs

b. Bill has at most two dogs

These truth-conditions account already for one of the clauses in the assertibility conditions listed in section §2, as the oddness of the follow-ups below signal.

(70) a. Bill has at least four dogs, #in fact he has three.

b. Bill has at most two dogs, #in fact he has four.

(71) a. Bill invited at least Sue and Liz, #in fact he only invited Liz.

b. Bill has at most Sue and Liz, #in fact he also invited Ed.

A consequence of this particular rendition of focus semantics for SMs is that the strength of a proposition containing an SM can only be assessed with respect to the focus semantic value of that proposition. The orderings induced by focus can be established both pragmatically or by the lexical properties of the focused constituents themselves. In the case of (65), the way in which professorships are ranked is set purely by pragmatics and world knowledge. In the case of numerals,

some, conjunctive plurals, etc., the ordering is set by the entailment properties of the lexical items themselves. An analysis of this type matches the predictions of those accounts that make use of a two Horn scale strategy (notably Mayr 2013, Kennedy 2015 and Schwarz 2016a)–modulo focus association–and extends it to virtually any constituent that SMs can associate with.

4.2.2 Step 2: Deriving alternatives

The main component of the analysis defended here is that the alternatives pertinent to the pragmatic calculus are generated by two independent mechanisms: focus alternatives and substitution by Horn scale-mates. The previous section illustrated one way to obtain a set of alternative proposition from the semantics of focus. We now can use this set of propositions to reason about plausible and more informative alternative propositions that the speaker could have uttered, just like we usually do in ordinary neo-Gricean pragmatics. I suggest that this constitutes the one set of relevant alternative propositions that is factored into the pragmatic calculus.

For instance, for the sentence *Sue is at least an assistant professor* in (65) above, the focus value provides the set of contextually restricted propositional alternatives in (66c). Utilizing the same focus alternatives we can reason about potential propositions that the speaker chose not to utter. For a proposition ϕ of the form [*at least p*], we calculate the relevant set of propositions by replacing the prejacent p by every proposition in $[\![\text{p}]\!]^{f_c}$.

(72) $Alt_{\text{FOC}}([\![65a]\!]) = \{[\![\text{at least } p]\!] \mid p \in [\![65a]\!]^{f_c}\} =$

$$\left\{ \begin{array}{l} \ldots \\ \textit{Sue is at least a visiting professor}, \\ \textit{Sue is at least an assistant professor}, \\ \textit{Sue is at least an associate professor,} \\ \ldots \end{array} \right\}$$

From the set (72) we identify the most informative alternatives, as in 73, since only stronger alternatives are relevant for Gricean reasoning. In the case of (65) the ordering is provided by world knowledge alone.[21]

(73) $SA_{\text{FOC}}([\![65a]\!]^{f_c}) = \left\{ \begin{array}{l} \textit{Sue is at least an associate professor,} \\ \textit{Sue is at least a full professor,} \\ \textit{Sue is at least a distiguished professor} \end{array} \right\}$

21 There are notions of informativity that could be applied here; in a Roberts (2012) style model of discourse, one could think of measuring informativity by keeping track of relevant propositions that address a particular question under discussion. Thus, *at least n+1* is more informative than *at least n* because the set of potential answers to the question under the discussion contains one less member. In this conception of informativity, the role of entailment is diminished.

All the propositions in (73) are such that they outrank the uttered proposition by virtue of instantiating a better alternative, i.e., a higher ranked professorship. This already looks like a set amenable to draw inferences from.

Focus alternatives are not the only way alternative propositions can be ordered, however. In the case of Horn scales, alternatives are ordered by virtue of the lexical properties of its scale-mates. This is no different in the case of SMs. As advanced before, I suggest that SMs participate in a Horn scale together with *only*. Traditional Horn scales like {*some, all*} or {*or, and*} are formed by sets of lexical items that stand in a relation of asymmetric entailment. Since SMs stand in an asymmetric entailment relation with *only*, this seems a plausible option. Moreover, the fact that *only*, like SMs, bears a conventionalized dependency on focus brings the connection between the two expressions closer.[22]

Thus, following the usual substitution method in neo-Gricean pragmatics (e.g., Sauerland 2004b) alternative propositions can be generated from the set of focus alternatives by swapping SMs with *only*.[23] In this case, we generate the new set of alternative propositions in (74) by trading *at least* for *only*, and then we pick those alternatives that asymmetrically entail the prejacent to generate a second set of Stronger Alternatives, as in (75).

(74) $Alt_{\text{HS}}([\![65a]\!]) = \{[\![\text{only } p]\!] \mid p \in [\![65a]\!]^{f_c}\} =$

$$\left\{\begin{array}{l} \ldots \\ \textit{Sue is only a visiting professor}, \\ \textit{Sue is only an assistant professor}, \\ \textit{Sue is only an associate professor}, \\ \ldots \end{array}\right\}$$

(75) $SA_{\text{HS}}([\![65a]\!]) = \left\{\begin{array}{l} \textit{Sue is only an assistant professor}, \\ \textit{Sue is only an associate professor}, \\ \textit{Sue is only a full professor}, \\ \ldots \end{array}\right\}$

22 For the purposes of this paper Horn scales have been chosen for their simplicity, but nothing in the analysis presented here hinges on this decision. In fact, there are obvious alternatives to the Horn scale system, which has been criticized in the past on the basis of being stipulative. For the reasons discussed in Katzir (2007), ultimately the substitution algorithm would have to be supplemented with some metric of complexity. The same results should obtain were we to adopt such elaborated approach.

23 One may wonder whether the presuppositional properties of *only* first discussed in the classic analysis of Horn (1969) could interfere with the implicature calculation mechanism. While I do not have space to address this worry here, other formulations of this idea are also possible. For instance, one could think of replacing *only* with the silent exhaustivity operator *Exh* (Fox (2007), a.o.), as Schwarz (2016a: fn.10) has pointed out. For now, assume Rooth's (1992) lexical entry for the exclusive *only*: $[\![\text{only}]\!] = \lambda C_{\langle st,t\rangle}.\lambda p_{\langle st\rangle}.\lambda w : \forall q[q \in C \wedge q(w) \leftrightarrow p = q]$.

Thus, Horn scales provide the second relevant set of alternatives that feeds the pragmatic calculus. Putting together both sets of Stronger Alternatives in (73) and (75), we get the final set of Stronger Alternatives over which we calculate implicatures.

(76) $SA([\![65a]\!]) = \left\{ \begin{array}{l} \textit{Sue is at least an associate professor,} \\ \textit{Sue is at least a full professor,} \\ \textit{Sue is at least a distiguished professor} \\ \textit{Sue is only an assistant professor}, \\ \textit{Sue is only an associate professor,} \\ \textit{Sue is only a full professor}, \\ \textit{Sue is only a distinguished professor} \end{array} \right\}$

4.2.3 Step 3: Calculating implicatures

Totally ordered associates The set of Stronger Alternatives calculated above in (76) is sufficient to derive the right kind of IIs simply by following the Gricean style reasoning about conversational cooperation laid out in section §4.1. We continue to use the same example as above in (65), now expressed differently, for simplicity.[24]

(77) $SA([\![65a]\!]) = \left\{ \begin{array}{l} [\geq \texttt{Assoc}], [\geq \texttt{Full}], [\geq \texttt{Dist}], \\ [O\ \texttt{Assis}], [O\ \texttt{Assoc}], [O\ \texttt{Full}], [O\ \texttt{Dist}] \end{array} \right\}$

Following the standard neo-Gricean practice, when first confronted with a utterance by her interlocutor, a listener usually assumes that the speaker is being cooperative, and so she deduces that the proposition must be true given the speaker's epistemic state; for the utterance (65), represented as $[\geq \texttt{Assis}]$, she deduces that $\mathsf{K}_S[\geq \texttt{Assis}]$ by Epistemic Implication. If there is no common understanding of the contrary, the listener may assume as well that the speaker is maximally informative–modulo relevance. Thus, if there are logically Stronger Alternatives that are both true and relevant, the speaker should have chosen one; since the speaker did not chose one of the Stronger Alternatives, it must be because she did not have sufficient grounds to claim so. Thus the listener is allowed infer that the speaker does not posses such knowledge, thereby deriving a set of Primary Implicatures, (78d).

24 The notational conventions are as follows: propositions are enclosed in square brackets, such that $[\phi]$ stands for some proposition containing a relevant expression ϕ, where ϕ informally represents the associate of the SM as a mnemonic of the relevant expression for the purpose of calculating implicatures. For instance, a sentence like *4 students came* is represented as $[\texttt{4}]$ and *Al and Mary came* as $[\texttt{A} \oplus \texttt{M}]$. With modifiers, $[\geq\ \phi]$ stands for $[\textit{at least}\ \phi]$, $[\leq\ \phi]$ for $[\textit{at most}\ \phi]$, and $[O\ \phi]$ for $[\textit{only}\ \phi]$.

(78) a. ASSERTION: $[\geq \texttt{Assis}]$
b. EPISTEMIC IMPLICATION: $\mathsf{K}_S[\geq \texttt{Assis}]$
c. $SA([\geq \texttt{Assis}])$ = (77)
d. PRIMARY IMPLICATURES:
$\neg\mathsf{K}_S[\geq \texttt{Assoc}] \wedge \neg\mathsf{K}_S[O\ \texttt{Assis}] \wedge \neg\mathsf{K}_S[O\ \texttt{Assoc}] \wedge \neg\mathsf{K}_S[O\ \texttt{Full}] \wedge \neg\mathsf{K}_S[O\ \texttt{Dist}]$

Since the negation of the weakest *at least* alternative from the Stronger Alternative set (77) entails the rest of the stronger *at least* alternatives, only the weakest one is factored into the computation. Together with the Epistemic Implication, these implicatures constitute the Implicature Base.

(79) IMPLICATURE BASE:
$\mathsf{K}_S[\geq \texttt{Assis}] \bigwedge \neg\mathsf{K}_S[\geq \texttt{Assoc}] \bigwedge$
$\neg\mathsf{K}_S[O\ \texttt{Assis}] \wedge \neg\mathsf{K}_S[O\ \texttt{Assoc}] \wedge \neg\mathsf{K}_S[O\ \texttt{Full}] \wedge \neg\mathsf{K}_S[O\ \texttt{Dist}]$

As in the case of disjunction we illustrated in section §4.1, nothing else is required from the listener to draw ignorance about the speaker's utterance, and at this point the task of deriving IIs from (79) is trivial: the Implicature Base entails that two and only two of the Stronger Alternatives in (77) must constitute epistemic possibilities for the speaker: $\neg\mathsf{K}_S\neg[\geq \texttt{Assoc}]$ and $\neg\mathsf{K}_S\neg[O\ \texttt{Assis}]$. This follows simply from the properties of the operators K and P and the fact that the associate of the SM is totally ordered.

To see why this is the case, consider first $\neg\mathsf{K}_S\neg[\geq \texttt{Assoc}]$. If $\neg\mathsf{K}_S\neg[\geq \texttt{Assoc}]$ were not true, $\mathsf{K}_S\neg[\geq \texttt{Assoc}]$, it would entail that $\mathsf{K}_S[O\ \texttt{Assis}]$ is true, given our assumption that $\mathsf{K}_S[\geq \texttt{Assis}]$ holds. But $\mathsf{K}_S[O\ \texttt{Assis}]$ directly contradicts the Primary Implicature that $\neg K_S[O\ \texttt{Assis}]$, rendering the Implicature Base inconsistent. Thus, it must be the case that $\neg\mathsf{K}_S\neg[\geq \texttt{Assoc}]$. A similar reasoning shows that the second entailment $\neg\mathsf{K}_S\neg[O\ \texttt{Assis}]$ also goes through. If $\neg\mathsf{K}_S\neg[O\ \texttt{Assis}]$ were not true, $\mathsf{K}_S\neg[\geq \texttt{Assoc}]$, it would entail that $\mathsf{K}_S[O\ \texttt{Assis}]$ is true, which contradicts the Primary Implicature that $\neg\mathsf{K}_S[O\ \texttt{Assis}]$. Thus the final step is simply to acknowledge that the Epistemic Entailment and the Primary Implicatures gang up together to generate a set of pragmatic entailments that are formally identical to the Signature of Ignorance we defined in (53) above.

(80) $\mathsf{I}_S[O\ \texttt{Assis}] \wedge \mathsf{I}_S[\geq \texttt{Assoc}]$

As a consequence, a speaker uttering a statement that contains an SM is providing quite precise in information about her epistemic state. No other epistemic possibilities are entailed. For instance, take the alternative proposition that *Sue is only an associate professor*, $[O\ \texttt{Assoc}]$. The listener will deduce a Primary Implicature of the form $\neg\mathsf{K}_S[O\ \texttt{Assoc}]$. In this case, the corresponding epistemic possibility, $\neg\mathsf{K}_S\neg[O\ \texttt{Assoc}]$, is a contingent statement, not entailed by the Implicature Base. In

fact, one could negate it, $\mathsf{K}_S\neg[O\ \texttt{Assoc}]$, without fear of contradicting any Primary Implicature or entailing any other relevant Stronger Alternative proposition. The epistemic state of a collaborative speaker who uttered (65) while being certain she is not an associate professor would look as follows:

(81) $\mathsf{K}_S[\geq \texttt{Assis}] \wedge \mathsf{K}_S\neg[O\ \texttt{Assoc}] \wedge \mathsf{I}_S[O\ \texttt{Assis}] \wedge \mathsf{I}_S[\geq \texttt{Assoc}]$

This accounts for the fact IIs of SMs only show partial ignorance: there two and only two epistemic possibilities that are necessarily calculated; the rest may but need not. This is, however, provided that the associate of the SM is totally ordered. We can now see why: these entailments are facilitated by a configuration where there are two Stronger Alternatives that jointly exhaust the space of possibilities denoted by the assertion. As a consequence, one or the other corresponding Stronger Alternative must be true, and so negating any one of them entails the truth of the other. Moreover, the analysis predicts that these results should obtain as well for any SM-stament where the associate of the SM is strictly ordered, like other Horn scales, evaluative scales, etc.

Finally, notice that these results track the assertibility condiditions of SMs as defined in section §2, and so we can now generalize them to totally ordered associates. (I only consider the case of *at least* here.)

(82) *Assertibility conditions of* at least *with total orders*:
A proposition ϕ of the form *at least n P* is assertible by a speaker *S iff*:

a. *S* knows that ϕ is true,

b. *exactly n P* is compatible with all *S* knows,

c. *more than n P* is compatible with all *S* knows, and

d. *S* knows that *less than n P* is not the case.

For an SM to be assertible, a collaborative speaker must meet certain "epistemic criteria". The first clause, (82a) is given by Epistemic Implication, and the last clause, (82d), follows from the truth-conditions of *at least*. The two middle clauses, (82b)/(82c) correspond to the epistemic possibilities entailed by the Implicature Base, and so they are to be observed. This accounts for the minimal conditions that speakers must meet to successfully use *at least* without implying unwarranted additional IIs, i.e., while still conveying partial ignorance.

Partially ordered associates Let us now turn to the cases where SMs associate with expressions that are only partially ordered. As we saw in section §2, the assertibility conditions of SMs in these cases differ from total orders in not requiring that the prejacent be an epistemic possibility. We see below how the present analysis

derives this difference. Consider the following sentence with *at least* associating with a conjunctive plural.

(83) a. Liz saw at least [Bill]$_F$

b. LF: [$_{S^1}$ at least C [$_{S^2}$ [$_{S^3}$ Liz saw [Bill]$_F$] ~ C]]

Assume a context with a reduced domain {*Bill, Sue, Ed*} of people that Liz could have seen. The derivation of the ordinary and focus semantic values proceeds as usual.

(84) a. $[\![65a]\!]^f = \{saw(Liz,x) \mid x \in D_e\}$

b. $[\![65a]\!]^o =$ *Liz saw Bill*

c. $[\![65a]\!]^{f_C} =$

$$\left\{ \begin{array}{l} \textit{Liz saw Bill, Liz saw Sue, Liz saw Ed,} \\ \textit{Liz saw Bill and Sue, Liz saw Bill and Ed, Liz saw Sue and Ed,} \\ \textit{Liz saw Bill and Sue and Ed} \end{array} \right\}$$

The truth-conditions are computed as in section §4.2.1, and as a result we obtain a lower bound on the range of allowable options, accounting for the oddness of (71) above. Then the derivation of alternatives and the calculation of implicatures proceeds exactly as in section §4.2.2 and section §4.2.3.

The crucial difference between SMs modifying partially ordered *vs.* totally ordered associates lies in the entailments of the Implicature Base. For instance, take Liz saw at least Bill and Sue. In such case, *at least* associates with *Bill and Sue*, which is an element of the salient plurality of individuals in the domain. This plurality, which takes the form of a join-semilattice (Link 1983), contains two pluralities that (i) are not comparable (i.e., not ordered with respect to each other), and that (ii) are minimally more informative than *Bill and Sue*: *Bill, Sue and Leah* and *Bill, Sue and Ed* (see Figure 1). As a consequence, the number of Stronger Alternatives that are minimally required to exhaust the space of possibilities denoted by the prejacent is no longer two, but three. For an assertion like *Sue is at least an assistant professor*, we carve out all the possibilities with two Stronger Alternatives, she is either an assistant professor or she holds a higher rank. But in order to exhaust all possibilities for *Liz saw at least Bill* in the previous situation, we need three: she either saw only Bill, or she saw at least Bill and Sue, or she saw at least Bill and Ed. This is illustrated in (85).

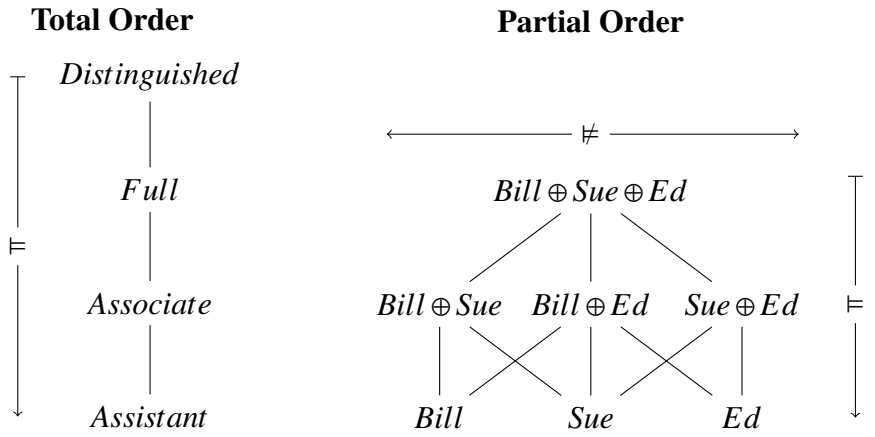

Figure 1 Entailment patterns of scales with different ordering properties

(85) a. $[\geq \texttt{Assist}] \nleftrightarrow [O\,\texttt{Assist}] \vee [\geq \texttt{Assoc}]$
b. $[\geq \texttt{B}] \nleftrightarrow [O\,\texttt{B}] \vee [\geq \texttt{B}\oplus\texttt{S}] \vee [\geq \texttt{B}\oplus\texttt{E}]$

This difference has major consequences when we compute the entailments of the Implicature Base. Again, consider *Liz saw at least Bill*. The calculation of the Implicature Base is summarized below.

(86) a. ASSERTION: $[\geq \texttt{B}]$
b. EPISTEMIC IMPLICATION: $\mathsf{K}_S[\geq \texttt{B}]$
c. $SA([\geq \texttt{B}]) =$
$\left\{ \begin{array}{l} [O\,\texttt{B}], \\ [O\,\texttt{B}\oplus\texttt{S}], [O\,\texttt{B}\oplus\texttt{E}], [O\,\texttt{B}\oplus\texttt{S}\oplus\texttt{E}], \\ [\geq \texttt{B}\oplus\texttt{S}], [\geq \texttt{B}\oplus\texttt{E}], [\geq \texttt{B}\oplus\texttt{S}\oplus\texttt{E}] \end{array} \right\}$
d. PRIMARY IMPLICATURES:
$\left\{ \begin{array}{l} \neg\mathsf{K}_S[O\,\texttt{B}] \wedge \\ \neg\mathsf{K}_S[O\,\texttt{B}\oplus\texttt{S}] \wedge \neg\mathsf{K}_S[O\,\texttt{B}\oplus\texttt{E}] \wedge \neg\mathsf{K}_S[O\,\texttt{B}\oplus\texttt{S}\oplus\texttt{E}] \wedge \\ \neg\mathsf{K}_S[\geq \texttt{B}\oplus\texttt{S}] \wedge \neg\mathsf{K}_S[\geq \texttt{B}\oplus\texttt{E}] \wedge \neg\mathsf{K}_S[\geq \texttt{B}\oplus\texttt{S}\oplus\texttt{E}] \end{array} \right\}$
e. IMPLICATURE BASE:
$\mathsf{K}_S[\geq \texttt{B}] \wedge \neg\mathsf{K}_S[O\,\texttt{B}] \wedge \neg\mathsf{K}_S[\geq \texttt{B}\oplus\texttt{S}] \wedge \neg\mathsf{K}_S[\geq \texttt{B}\oplus\texttt{E}]$

Unlike with totally ordered associates, the resulting Implicature does not entail that any one of the Stronger Alternatives constitutes an epistemic possibility for the speaker. Indeed, now we could negate any Stronger Alternative without contradicting nor entailing the truth of any other. Thus, suppose that the speaker knew that it is not the case that Sue saw only Bill, $\mathsf{K}_S\neg[O\,\texttt{B}]$. Conjoining this assumption with the Implicature Base results in a contingent set of propositions: all it says is that the

speaker knows that Sue saw somebody else besides Bill, but she does not know who exactly.

(87) $\mathsf{K}_S[\geq \mathtt{B}] \wedge \mathsf{K}_S\neg[O\ \mathtt{B}] \wedge \neg\mathsf{K}_S[O\ \mathtt{B}] \wedge \neg\mathsf{K}_S[\geq \mathtt{B}\oplus\mathtt{S}] \wedge \neg K_S[\geq \mathtt{B}\oplus\mathtt{E}]$

Knowing that $\mathsf{K}_S\neg[O\ \mathtt{B}]$ does not settle the question as to which one of $[\geq\ \mathtt{B}\oplus\mathtt{S}]$ or $[\geq\ \mathtt{B}\oplus\mathtt{E}]$ might be true, and so the speaker is predicted to be ignorant precisely about these two Stronger Alternatives: since together they carve out the remaining space of possibilities, negating one of them, e.g., $\mathsf{K}_S\neg[\mathtt{B}\oplus\mathtt{S}]$ would in turn entail the truth of the second, $\mathsf{K}_S[\mathtt{B}\oplus\mathtt{E}]$, contradicting once again the corresponding Primary Implicature that $\neg\mathsf{K}_S[\mathtt{B}\oplus\mathtt{E}]$ and resulting in an inconsistent set of beliefs. In these situations, all the speaker is allowed to infer is an epistemic state of the following form.

(88) $\mathsf{K}_S[\geq \mathtt{B}] \wedge \mathsf{K}_S\neg[O\ \mathtt{B}] \wedge \mathsf{I}_S[\geq \mathtt{B}\oplus\mathtt{S}] \wedge \mathsf{I}_S[\geq \mathtt{B}\oplus\mathtt{E}]$

The allowed inferences correspond again to the assertibility conditions of SMs associating with conjunctive plural, repeated below from (35).

(35) *Assertibility conditions of* at least *with partial orders*:
For a domain D $\{a, b, c, d\}$, a proposition ϕ of the form *at least* $a\oplus b$ P is assertible by a speaker S *iff*:

a. S knows that ϕ is true,

b. there is some x in D such that $a\oplus b\oplus x$ P is compatible with all S knows,

c. S knows that no x part of $a\oplus b$ is such that *only* x P.

The first and last clauses follow from the principle of Epistemic Implication and the truth-conditions of *at least*, respectively. The middle clause corresponds to the epistemic possibilities entailed by the Implicature Base, which in this case amount solely to the existence of some possibly unidentifiable Stronger Alternative that is compatible with all the speaker knows. As before, these constitute the minimal conditions that speakers must meet to successfully use *at least* when modifying a partially ordered associate without implying unwarranted additional IIs.

The "epistemic criteria" in these cases are weaker than with totally ordered associates: the exahustive interpretation of the prejacent, need no longer be an obligatory epistemic possibility for the speaker. (The reader is invited to check that, *mutatis mutandis*, the same results are obtained for the SM *at most*.)

5 Discussion

5.1 Assessment

The goal of the analysis defended above was to account for the properties of the IIs described in section §2. In order to do so, it is argued that it suffices with (i) a basic epistemic logic and (ii) the assumption that we need to factor in alternatives generated from two different sources, focus alternatives and Horn scales (cf. Mayr 2013, Kennedy 2015, Schwarz 2016a). As shown in section §3, there is independent evidence showing that IIs of SMs are conversational implicatures (in the sense of Grice 1975 and Gazdar 1979) and that they show a lexical dependency on focus placement (in the sense of Beaver & Clark 2008). Taken together, these ingredients allowed us to analyze SMs in a neo-Gricean fashion and account for the properties of the IIs described in section §2. In what follows I summarize and briefly discuss the merits of the analysis.

Flexibility One of the most obvious advantages of the present analysis is that it provides a uniform treatment to all cases where SMs are allowed. The reasoning process that leads to IIs is a general pragmatic mechanism triggered by external factors like conversational efficiency and speaker-hearer cooperation, and so the underlying mechanisms for calculating IIs across associate types are kept constant, no extra assumptions are required. This is greatly facilitated by appealing to and reusing focus alternatives, making it possible for SMs to refer to sets of expressions that would otherwise not be naturally ordered, thereby failing to constitute a scale.

Strength of Ignorance As Nouwen (2015) and Schwarz (2016a) already noted, the epistemic inferences that can be drawn from SM-statements are somewhat weaker than those of disjunction. A disjunctive statement conveys IIs about each one of the individual disjuncts, a property referred to as total ignorance. Instead, SMs do not convey ignorance about all the individual values in the range of the SM that the speaker could be ignorant about. As argued in section §2, Büring's (2007) original idea that IIs of SMs are better characterized as a simple disjunction should be generalized to the "simplest" disjunction possible to accommodate conjunctive plurals. The analysis provided here is able to calculate IIs that are equivalent to the simplest disjunction possible, thereby accounting for the fact that SMs only convey partial ignorance, while still being compatible with total ignorance.

Assertibility conditions As shown in the previous section, the assertibility conditions of SMs are no longer surprising under the present analysis. The account defended here provides a precise definition of the "epistemic requirements" that

speakers must meet in order to successfully use an SM-expression–under the assumption that we are not dealing with a conversational situation where some pragmatic principle is being obviously disregarded. The assertibility conditions of SMs are kept constant and derived via a general schema of implicature calculation. However, it was shown that different factors may affect the assertibility conditions of SMs in certain cases. Three such main factors were mentioned above, namely focus placement, pragmatic assumptions about the discourse, and the ordering of the SMs' associate. Accordingly, the paper shows that the assertibility conditions of SMs vary in predictable ways when there is a change in the status of any one of these factors.

Two of these three factors are accounted for by assuming (i) obligatory focus association and (ii) that IIs arise as conversational implicatures. As it was shown in section §2, focus placement determines what the IIs that the SM is conveying are about. Put it otherwise, it is not possible for an SM-statement with a focused phrase XP_F to convey an II about some other phrase YP in the sentence. Conversely, in the absence of a focused phrase in the sentence, no IIs arise. The analysis defended here handles this dependency by relying on focus alternatives to generate a set of alternative propositions that are later factored into the pragmatic calculus. This assumption was defended in section §3.2 above.

Absence of focus is not the only way to waive the requirement of computing IIs with SMs. IIs are pragmatic implicatures that show a strong context-dependency and do not arise across all types of conversational situations. This accounts for the second of three factors that can have an impact on the assertibility conditions of SMs. In particular, IIs depend crucially on the common understanding among the participants in the conversation that the speaker is being maximally informative. Conversational situations where the Maxim of Quantity is predicted to be suspended are shown to correlate with a lack of IIs. The analysis can correctly capture such context-dependency by relying on the Maxim of Quantity, which can be easily suspended. The assumption that IIs arise a conversational implicatures is defended in section §3.1.

Non-uniformity The present analysis predicts that the status of the IIs differ across different types of associates, thus deriving the patterns of non-uniformity observed in section §2. The exhaustive interpretation of the prejacent necessarily constitutes an epistemic possibility for the speaker only if the associate of the SM is totally ordered. If the associate is partially ordered, this possibility is merely that, a possibility, and it is not required to figure under the options that the speakers has to consider mandatorily. The analysis presented in this paper is able to capture this point of variation between the numeral case and some phrasal associates of SMs without further stipulations. The ordering of the SM's associate constitutes the third

and final factor that might have an impact on the assertibility conditions of SMs, and no extra assumption are required to derive it.

5.2 Comparison to other approaches

The analysis improves on previous approaches in a number of respects. The neo-Gricean analysis defended here replicates the results obtained by Büring (2007) by deriving IIs in a principled way, without appealing to disjunctive definitions of SMs. In doing so, the present account is in line with a recent proposal by Schwarz (2016a). The analysis also accounts for the properties of IIs described in section §2 which, unlike in Schwarz's (2016a) treatment, also consider cases where SMs modify non-numeral scales, making explicit the connection between SMs and focus on the way.

Despite the abundance of work on the numeral case, only a handful of papers have focused on providing a generalized treatment that extends to all cases where SMs can appear (see for instance Nouwen 2010, Mayr 2013, Kennedy 2015, Nouwen 2015, Cummins 2015, Spychalska 2015, Schwarz 2016a, to name some recent papers focusing only on numerals). Given the similarities in the kind of epistemic inferences that obtain from numerals and non-numerals scales, I consider the present analysis to be a significant improvement over those with a lesser empirical coverage. In what follows I comment on how some of the extant accounts relate to mine with respect to the aspects of IIs mentioned in the previous section. (I consider only those papers which do provide an extension to the non-numeral cases.)

5.2.1 The connection to disjunction

Building on Krifka (1999), Büring (2007) pointed out that the IIs of *at least* are equivalent to the disjunctive statements as as in (89):[25]

(89) $[\![$at least *n* P Q$]\!]$ = $[\![$[exactly *n* P Q] or [more than *n* P Q]$]\!]$

Büring (2007) provided an insightful observation about what part of the meaning of *at least* amounts to, but the analysis he provided has been criticized since, mostly on conceptual grounds. Concretely, the most commonly found criticism is that the semantic equality between *at least* and the disjunction in (89) is unwarranted (see e.g., Geurts & Nouwen 2007, Coppock & Brochhagen 2013b). For instance, having *or* in the meta-language may very well be sufficient to assure that *at least*

25 Once the full range of data is taken into account, it seems that Büring's (2007) intuition should be rephrased as requiring that SM expressions be equivalent to the simplest disjunction that carves out all the space of possibilities denoted by the assertion. This would effectively take care of the facts discussed about the behavior of partially ordered associates.

will behave like a disjunction at some level. However, as Coppock & Brochhagen (2013b) point out, the fact that some proposition ϕ may be described as a disjunction in the meta-language does not seem to clarify the question as to why we can then draw implicatures from this disjunction. The reasoning would be like this: if a speaker *S* utters expression ϕ, such that the meta-language description of ϕ contains a disjunctive symbol, then *S* considers each disjunct of the meta-language description of ϕ to be possible. There is an obvious problem with this: the reasoning assumes speaker's knowledge (conscious of unconscious) of the meta-language description used to describe the meaning of ϕ. But this is rather implausible, how could speakers have access to what semanticists think is the best way to describe a linguistic phenomenon?

Sentences containing *at least* are not disjunctions on the surface level, and there is no evidence for a syntactic transformation to turn *at least* statements into disjunctions at LF. And yet, even if SMs are not disjunctive statements, giving up (89) requires to replace it with some other calculus that successfully derive IIs of SMs, without necessarily taking SMs to be disjunctions at any level. Call this "Büring's challenge": to find the best way to derive the disjunctive-like behavior of SMs.

Since then, various forms of meeting this challenge have been proposed (see Mayr 2013, Coppock & Brochhagen 2013b and Schwarz 2016a). This paper follows the strategy best spelled out in Schwarz (2016a), and argues that the resemblance to a disjunctive statement follows if we carefully choose the right pieces from other relatively well-studied phenomena and put them together in the right way.

However, there have been attempts at deriving the disjunction-like behavior of SMs that depart substantially from the one defended here. I discuss two such approaches below and show that even if they may succeed in meeting Büring's challenge, their analyses fail to derive some of the other properties of SMs that were discussed in section §2.

5.2.2 Geurts & Nouwen (2007) and Geurts et al. (2010)

Geurts & Nouwen (2007) and Geurts et al. (2010) adopt a modal analysis for SMs where, for instance, *at least n* means *certainly n and possibly more*. IIs are taken to be derived pragmatically, since *possible* may imply *not certain*. Given their lexical entries for SMs, the IIs they predict are exactly the ones we have defended here for the case of totally ordered associates.[26]

26 Cummins & Katsos (2010) show that these truth-conditions are problematic when SMs occur in certain embedded positions. Since those contexts fall out of the scope of this paper, I will not comment on them.

(90) a. $[\![$At least n A's are B$]\!]$
$= \Box\exists x[A(x) \wedge |x| = n \wedge B(x)] \bigwedge \Diamond\exists x[A(x) \wedge |x| > n \wedge B(x)]$

b. $[\![$At most n A's are B$]\!]$
$= \Diamond\exists x[A(x) \wedge |x| = n \wedge B(x)] \bigwedge \neg\Diamond\exists x[A(x) \wedge |x| > n \wedge B(x)]$

As the authors observe, applying the lexical entry (90a) to a sentence like *Sue won at least a bronze medal* results in a contradiction: it would entail the necessity that Sue won a bronze medal, and the possibility that Sue won some other more valuable metal. In order to extend the analysis to non-entailing scales, the authors provide a second lexical entry for SMs. (I only consider *at least* here.)

(91) Given a proposition α and a set of alternative propositions of α, $[\alpha]_A$ ordered according to some salient order $\trianglelefteq$ of alternatives (ordered by pragmatic strength), $[\![\text{AT LEAST } \alpha]\!] = \exists\beta[\alpha \trianglelefteq \beta \wedge \Box\beta] \wedge \exists\gamma[\alpha \trianglelefteq \gamma \wedge \Diamond\gamma]$.

An interpretation along the lines of (91) however provides the wrong kind of IIs. As it is familiar from the literature on Free Choice disjunction, statements of the form $\Box(p \vee q)$ trigger the inference that every disjunct must be a possibility. Thus, including a disjunctive statement as part of the semantic import of SMs makes it equivalent in the relevant respect to an ordinary disjunction. As a consequence, we expect that after hearing a sentence like *Sue won at least a bronze medal*, the addressee will infer that Sue could have won any of the three medals. This is total ignorance.

(92) a. $\Box[win(S, bronze) \vee win(S, silver) \vee win(S, gold)]$

b. $[\![(92a)]\!] \rightsquigarrow \Diamond[win(S, bronze) \wedge win(S, silver) \wedge win(S, gold)]$

Given the semantics in (91) and the truth conditions in (92a), (92b) is an inference that goes through independently of the epistemic state of the agents involved in the conversation. But this is total ignorance, and we already saw in section §2 that SMs convey weaker partial IIs. This behavior cannot be captured by a straight analogy with disjunction.

Besides providing IIs that are stronger than attested, in general–but especially after the revision in (91)–it is not clear how to account for the IIs of SMs and their context dependency under the modal account of SMs. If IIs of SMs are the fruit of reasoning over modal statements like (92b), we should expect them to pattern like formally similar constructions, like Free Choice disjunction in this case, and yet we do not observe this context dependency in the latter case.

5.2.3 Coppock & Brochhagen (2013a,b)

Coppock & Brochhagen (2013b) provide a theory of SMs couched within Inquisitive Semantics. In this proposal it is assumed that SMs share with other epistemic operators the property of being *interactive*: SMs require that there be at least two epistemic possibilities that are compatible with the speaker's knowledge.

In Inquisitive Semantics, denotations are represented as sets of possible worlds (possibilities) corresponding to the set of possible answers to the QUD. The set of available possibilities is then further constrained by the information state of the speaker, that is, by the set of possible worlds epistemically accessible to the speaker. For instance, assuming that k_s represents the information set of the speaker, a proposition p restricted to k_s is expressed as the set $\{p' \mid \exists q \in p : p' = k_s \cap q\}$. In order to ensure that k_s restricts p in the right way, the authors propose an additional pragmatic principle to derive IIs.[27]

(93) MAXIM OF INTERACTIVE SINCERITY:
If p is interactive, then p is interactive in the speaker's information set.

By assumption SMs are interactive, and so the denotation of p as restricted to k_s must contain more than one possibility. The principle in (93) enforces this requirement. Consider for instance a reduced domain {*Ann, Bill*} and suppose that the speaker knows that only Ann snored and Bill did not. In this situation a sentence like *At least Ann snores* is odd or misleading at best. The denotation of such sentence would amounts to the set of sets of worlds that verify it, that is, $\{\{[A \oplus B], [O\ A]\}, \{[O\ A]\}\}$. The speaker information set is smaller in this case, it contains the singleton $\{[O\ A]\}$, the set of worlds where only Ann snores. If we restrict the set of propositions in p with those in k_s, only $\{[O\ A]\}$ survives:

(94) $\{p' \mid \exists q \in \{\{[A \oplus B], [O\ A]\}, \{[O\ A]\}\} : p' = \{[O\ A]\} \cap q\} \quad \leftrightarrow \{[O\ A]\}$

Since this proposition is not interactive, the maxim of Interactive Sincerity is violated, and the sentence is predicted to be odd. It follows that in the case of *at least* sentences there must be some Stronger Alternative to the prejacent that is consistent with the speaker's information set. The authors do not explicitly discuss how exactly to derive these Ignorance Inferences, this could be done by virtue of SMs' signaling that the speaker is unable to restrict her epistemic state to a singleton (cf. Alonso-Ovalle & Menéndez-Benito 2010 for epistemic indefinites).

While Coppock & Brochhagen's (2013b) account correctly predicts partial ignorance, it does so by providing weaker IIs than required. Notice that all that it is required of SMs is that they denote any two possibilities, but nothing is said about

27 Schwarz (2016b) provides a thorough criticism of the Maxim of Interactive Sincerity, so here I focus solely on those aspects relevant to SMs' IIs.

which possibilities. As a consequence, no particular proposition is required to be in the information state of the speaker when she utters an SM-proposition. But, as we learned in section §2, there are certain propositions about which the speaker must be ignorant about; these are the PREDICTABLE IIs. One such case is the exhaustive interpretation of the prejacent when SMs modify totally ordered scales. Thus, their account predicts that there should be nothing odd about (26a), as long there are two other options that the speaker is considering.

(26a) #Bill ate at least two apples but I know that he didn't eat just two.

For instance, if the speaker knows that Bill ate exactly three or four apple, there are two possibilities alive for the speaker, hence Interactive Sincerity is observed and no oddness is predicted, contrary to intuitions.[28]

In order to fix this problem, the authors introduced an additional pragmatic principle in Coppock & Brochhagen (2013a).

(95) MAXIM OF DEPICTIVE SINCERITY:
If p highlights a possibility q, then the speaker considers q possible.

Depictive Sincerity rests on the notion of *highlighting*: a possibility q is highlighted if it is overtly expressed. Assuming that propositions of the form *SM n* overtly express the prejacent, speakers uttering a sentence like *Bill ate at least two apples* must consider the possibility that Bill ate two apples. But now if we extend Depictive Sincerity to the conjunctive case, *At least Ann snores*, that only Ann snored must be considered an epistemic possibility for the speaker, contrary to fact.[29] As a consequence of Depictive Sincerity, the prejacent of an SM-statement must always constitute an epistemic possibility for the speaker. Unlike Interactive Sincerity alone, Depictive Sincerity enforces this requirement and hard-wires it in the form of a pragmatic constraint regulating the use of SMs.[30] Thus, the inquisitive approach provides IIs that are either too weak or too strong and, at any rate, cannot account for the difference observed between totally and partially ordered associates.

Moreover, by relying on general mechanisms that bear their own independent relationship to the current discourse (like focus and Grice's Maxims based on conversational goals), theories like the one defended here are able to accommodate more easily such the context-dependency of SMs' IIs. In contrast, theories relying

28 Experiments conducted by the same authors in Coppock & Brochhagen (2013a) confirmed that these predictions are indeed problematic.

29 See section §2 for discussion and Mendia (2016a) for experimental evidence in support of this observation.

30 Notice that what is required of SMs is that the prejacent is true, leaving open the question as to whether it has to be exhaustively true. For the purposes of this paper, I will assume that it is indeed the exhaustive interpretation of the prejacent what is required to be an epistemic possibility; otherwise its application would be vacuous in some cases (e.g., *at least Ann snored*).

on pragmatic principles that are contextually underspecified, like the Maxim of Interactive Sincerity and the Maxim of Depictive Sincerity, cannot account for the context-dependency of Ignorance without further ado.

5.3 Open issues and final remarks

This section touches upon a number of issues and questions that did not receive the deserving attention in the course of the explanation.

5.3.1 What is with *at most*?

The analysis presented here is intended to explain the properties of both *at least* and *at most*, and is thus based on an expectation that all things being equal, *at least* and *at most* behave alike except in their monotonic properties. Recall that a key motivation for the present analysis was the observation that total and partial orders differ in the IIs conveyed. More concretely, when uttering an SM-statement with a partially ordered associate, the prejacent need not be one of the possibilities that the speaker considers. The experiment reported in Mendia (2016b) substantiated this claim in the case of *at least*. However, the experiment failed to find the same results for *at most*: although, numerically, the results show an improvement when negating the prejacent of an *at most* statement with partially ordered associates over negating the prejacent with totally ordered associates, the results were not significant.

The asymmetric experimental results raise the possibility that we got the intuitions wrong for *at most*. Below is the same scenario used for *at least* in section §2, minimally modified for the SM *at most*.

(96) **Context**: Sherlock Holmes went on vacation for a couple of days and let some his friends celebrate a dinner on 221B Baker Street: Dr. Watson, Mrs. Hudson, Mycroft, Irene Adler and some of the Baker Street Irregulars. After vacation, he returns to his room only to discover that somebody has been messing with his chemistry set. Inspector Lestrade of Scotland Yard is with him, and asks:

(97) IL. Who do you think that touched the chemistry set?
SH. It was at most Mycroft, Mrs. Hudson, and two of the irregulars, but not all of them touched it.

Sherlock's answer in (97) is meant to support an epistemic state where he knows for a fact that each member of a selected group of suspects could have touched his chemistry set, and yet not all of them did touch it, although maybe more than one of them did. In my own and my informants' judgment, this is fine sentence given the

depicted scenario. Moreover, there is a clear contrast with the variant with a totally ordered associate of *at most* in (98), where Sherlock's answer is plainly odd.

(98) IL. How many people do you think that touched the chemistry set?
SH. #It was at most two people, but not exactly two.

I would like to contend that our intuitions in (96)–(98) are correct, and point out a couple of reasons why Mendia (2016b) failed to find support for these intuitions. For one, it seems difficult to extract definitive conclusions about *at most* from the results of the experiment alone due to a potential methodological issue in the experiment. In the experimental paradigm used, there was no perfect match of the *at least* stimuli for *at most* when testing for statements where the prejacent was being negated: while stimuli for the critical *at least* cases looked like *at least XP but not only XP*, the corresponding *at most XP but not only XP* is not a viable option. Thus, there is no *only* equivalent for *at most*. That is, while a speaker uttering a sentence of the form [*at least* ϕ] could consider [*only* ϕ] as both a grammatical and plausible alternative to her utterance, this is not the case for *at most*. In an ordinary *at most* sentence like *at most Bill and Al came*, the sentence *only Bill and Al came* would never figure as a plausible alternative to the former. Mendia (2016b) suggests that this might be due to the presuppositional content of *only* itself, or due to the entailment properties of *at least* versus *at most*: in many theories of *only*, [*only* ϕ] requires a higher ranked alternative that does not hold, thereby bringing an existential commitment about the denied higher ranked alternative. On the other hand, [*at most* ϕ], which we saw to be contributing universal quantification over alternative propositions, does not (see discussion in Coppock & Beaver 2011). This property of *only* makes it problematic to find a context where both [*at most* ϕ] and [*only* ϕ] are felicitous, which is precisely the key data-point required to assess the reported results in the critical conditions. For the same reasons, one may wonder whether the proposed Horn scale {*at most, only*} is a feasible option at all. In so far the intuitions reported above in (96)–(98) are correct, the predicted results could be replicated without using Horn scales in an analysis where the relevant alternatives are determined on the basis of structural considerations (as in Katzir 2007; see footnote 22).

The aforementioned methodological issues aside, Mendia's (2016b) results reveal a host of environments where *at most*, but not *at least*, misbehaves, including unexpected semantic effects related to interpreting the upper bound (see as well Coppock & Brochhagen 2013b and Cummins 2015). These findings may suggest that there is a greater divergence between *at least* and *at most* than previously thought (cf. Penka 2015). A fruitful avenue for future research is to carefully investigate whether this is in fact the case, and if so, what it is that makes the two SMs behave differently.

5.3.2 Scalar implicatures

An important aspect of SMs that the paper has not touched upon is their lack of scalar implicatures. Consider the examples in (2) again: in neither case do we generate the corresponding scalar implicature.

(2) a. Bill has at most two daughters.
↛ It is not the case that Bill at most one daugter

b. That caterpillar has at least twenty legs.
↛ It is not the case that the caterpillas has at least twenty-one legs

Any successful account of SMs should derive the fact that such strengthening is blocked. In Gricean frameworks, scalar implicatures require the extra assumption that the speaker is maximally knowledgeable about the question that the proposition she is uttering is making a contribution to. That is, for a Stronger Alternative ϕ by S, either $\mathsf{K}_S\phi$ or $\mathsf{K}_S\neg\phi$, in so far as the result is consistent with the assertion and the Primary Implicatures (Sauerland 2004b, van Rooij & Schulz 2004). This assumption is commonly referred to as the "epistemic step" (also the "competence" or "authority" assumption).

With respect to SMs, both Mayr (2013) and Schwarz (2016a) show that a double Horn-scale strategy in a neo-Gricean analysis of scalar implicatures would deliver the wrong results. Let us take the numeral case as an illustration: the analysis presented above only generates IIs about two of the Stronger Alternatives. In principle, then, any other additional alternative could be strengthened by the epistemic step to a stronger (scalar) type of inference. That is, for any one Stronger Alternative ϕ for which the system fails to generate the possibility implication (i.e., $\neg\mathsf{K}_S\neg[\phi]$), ϕ could be strengthened to $\mathsf{K}_S\neg[\phi]$. This is problematic since, in Mayr's (2013) assessment, a sentence like *at least two people came*, is then predicted to convey that exactly two people came or exactly three people came, which is clearly an unavailable interpretation.

While this paper does not directly address this issue, one can think of two ways of sidestepping the problem of unwanted implicatures and inconsistency while retaining the basis tenets of the paper. The first and most straightforward is to fully embrace Schwarz's (2016a) suggestion that the neo-Gricean calculus should incorporate a mechanism that preserves consistency during the derivation of implicatures that is more advanced than that of Sauerland (2004b). The author develops this idea by adapting the notion of "Innocent Exclusion", familiar from grammatical approaches to scalar implicatures (Fox 2007, Chierchia et al. 2012) to the neo-Gricean camp.[31]

31 I refer the reader to Schwarz (2016a) for details. A full assessment of whether this solution would forestall the derivation of unwanted implicatures while preserving consistency in the context of the present account is left for future research.

A second option, which is arguably more theoretically involved and roundabout, is to deny altogether that scalar implicatures should be explicated in a Gricean framework. In the recent past, more and more evidence has been marshaled in favor of the view that scalar implicatures should be derived by means of grammatical operators (Fox 2007, Chierchia et al. 2012). To date, the most compelling empirical argument for this family of "grammatical" approaches to implicatures is the existence of embedded scalar implicatures. This fact alone seems to render the Gricean approach as a non-starter, since in a Gricean framework the implicata are always calculated at the propositional level (but see Geurts 2010 for a suggestion to derive embedded implicatures *à la* Grice). Primary Implicatures, and so Ignorance Inferences as well, cannot be embedded, however. For instance, (99a) can only be contradiction free under an exclusive interpretation of the disjunction, which is usually attributed to a scalar implicature. As Sauerland (2004a) points out, if Primary Implicatures were embeddable, (99b) should be able to convey something like this: when the speaker is sure that John or Mary worked and not sure that John worked, the result was good, but when the speaker is sure that John worked, the result was a mess. This is clearly not an available reading of (99b) (examples from Sauerland 2004a).

(99) a. When John or Mary worked, the result was good, but when John and Mary worked, the result was a mess.

b. #When John or Mary worked, the result was good, but when John worked, the result was a mess.

The resulting picture would be one where we are Gricean when it comes about calculating Primary Implicatures and we stop there. The reasoning could go as follows. First, notice that the epistemic step is nowhere to be found in Grice's system (it does not follow from Grice's formulation of the maxims of Quantity). The listener, however, is free to make the assumption that her interlocutor is an expert on the subject matter. Since the empirical result of making such assumption is a family of implicatures that can be embeddable, the strengthening of the speaker's utterance must happen by means that allow embedding, for instance by invoking an exahustivity operator, with a semantic meaning similar to *only*. In turn, the exhaustivity operator comes well equipped to avoid inconsistencies, and so one can remain purely Gricean, albeit only for the purposes of calculating Primary Implicatures. This would effectively remove the need to replicate the merits of grammatical approaches to scalar implicatures in the Gricean framework. To assess the plausibility of this line of thought, however, involves a much more in-depth examination that I have space for here.

5.3.3 Embedding contexts

This paper has nothing new to say about the effects arisings as a consequence of embedding SMs. As Büring (2007) already noted, IIs of *at least* can be obviated in certain contexts, most notably when it appears embedded under universal quantifiers.

(100) Every student wrote at least two papers.

While (100) might still convey ignorance of the speaker as to what the exact requirement is, the most natural reading is that where the number of books required to get an A is either irrelevant or known to the speaker. In this sense, a speaker can use (100) to convey a minimal requirement reading in addition to a state of ignorance. This is the so-called "authoritative" reading of SMs.[32] The present analysis derives the lack of IIs for free (see also Schwarz & Shimoyama 2011). Take $\forall[\geq 2]$ to represent the meaning of (100), and $\mathsf{K}_S\forall[\geq 2]$ as the inference resulting from Epistemic Implication. In this system, IIs are derived by virtue of finding a pair of Stronger Alternatives that jointly exhaust the space of possibilities covered by the utterance. Note, however, that $\forall[=2]$ and $\forall[\geq 3]$ fail to cover such space, since $\forall[\geq 2]$ could be true by virtue of some students writing exactly two papers while other students wrote more than two, in which case neither $\forall[=2]$ or $\forall[\geq 3]$ would be true. As a consequence, none of the corresponding possibility implicatures are entailed, $\neg\mathsf{K}_S\neg\forall[=2]$ and $\neg\mathsf{K}_S\neg\forall[\geq 3]$, and no IIs are predicted.

What the neo-Gricean analysis cannot account for are the authoritative readings of *at most* under possibility modals.

(101) Bill can invite at most five friends to the party.

The sentence in (101) might convey that the speaker knows that the exact number of friends Bill is allowed to invite ranges from zero to five; as long as he does not invite more than five, he will adhere to the law. I do not have a solution to this problem. Other analyses fare no better. For instance, Geurts & Nouwen (2007) propose a solution in of the epistemic/authoritative dichotomy in terms of modal concord, but fatal issues with their general approach were pointed out in section §5.2.2. Kennedy's (2015) analysis of numerals in terms of second-order properties of degrees can account for the authoritative reading above, but it is not obvious how to extend such an analysis to the phrasal cases of SMs. Similar objections can be

32 In the case of *at least*, IIs can be obviated in contexts that have been argued to involve some sort of universal quantification, like modal verbs, generics and imperatives.

(i) a. Bill {must/has to/is required to} read at least two papers to get an A.
b. Spiders have at least two eyes.
c. Calculate at least one root of the equation $8x^5 - 6x^4 - 83x^2 - 6x + 8 = 0$.

raised against Nouwen (2010) and Penka (2015). I leave open the question as to whether any of these analyses could be amended to account for the phrasal cases.

6 Summary

This papers accomplishes two things. It provides a close examination of the Ignorance Inferences (IIs) conveyed by Superlative Modifiers (SMs), and establishes that SMs convey partial ignorance, as opposed total ignorance. We then turn to the question of what, if anything, speakers must be ignorant about. Specifically, we ask two questions about the nature of these inferences, which we dubbed PREDICTABILITY and UNIFORMITY.

(6) PREDICTABILITY
Is there any proposition in particular about which the speaker must be ignorant about so that she can successfully use an SM?

(11) UNIFORMITY
Are the inferences that come with SMs the same across the board, regardless of the associate type of the SM?

The answer to the PREDICTABILITY question is positive: in order to felicitously use SMs, speakers must meet certain epistemic conditions. In turn, these conditions correlate with what is minimally predictable about the speaker's epistemic state after she utters an SM-statement. With respect to the UNIFORMITY question, it is shown that the answer should be negative: SMs modifying totally ordered associates trigger IIs that are formally different than those of SMs modifying partially ordered associates. The difference lies in what is required of the exhaustive interpretation of the prejacent: the speaker **must** necessarily take this epistemic possibility into account with totally ordered scales, but she **need not** do so with partially ordered associates.

The second part of the paper develops an analysis of IIs of SMs that uniformly accounts for both numeral and phrasal cases. The account, based on a basic epistemic logic, defends the idea that we need to factor in alternatives generated from two different mechanisms and makes the novel claim that one such mechanism is focus alternatives. The analysis improves on previous approaches in a number of respects. It replicates the results obtained by Büring (2007) by deriving IIs in a principled way, without appealing to any form of special status for Superlative Modifiers. In addition, the analysis improves on Schwarz (2016a) in that it is extended to *at most* and to a variety of scales other than the numeral, thus covering a wider empirical range.

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