

BLURRED CONDITIONALS*

1. What is the meaning of *if . . . then* clauses? More than 2000 year ago, Philo argued for what we now call *material implication*. And Chrysippos fought for what we now call *strict implication*. In modern times, G. Frege argued for material implication again. And so did C. I. Lewis for strict implication. Philo and Chrysippos, Frege and Lewis defended their ideas, found followers and died.

Why could this happen? Do people differ in their interpretations of sentences in such a way that they can't be reconciled? I'd better be fair to Frege. He didn't aim at giving an analysis to *if . . . then* clauses in natural language. Consider his remarks concerning material implication:¹

The thought expressed by the compound sentence

"If I own a cock which has laid eggs today, then the Cologne cathedral will collapse tomorrow morning."

is also true. Someone will perhaps say: "But here the antecedent has no inner connection at all with the consequent." In my account, however, I required no such connection and I ask that "if *B*, then *A*" should be understood solely in terms of what I have said and expressed in the form "not ((not *A*) and *B*)". It must be admitted that this conception of a hypothetical compound thought will at first be thought strange. But my account is not designed to keep in step with ordinary linguistic usage, which is generally too blurred and variable for the purposes of logic.

But then, do scientists and ordinary people differ in their interpretations of sentences in a crucial way? Do our semantic rules change fundamentally as soon as we talk as scientists?

In 1689, John Wallis showed how hypothetical judgements can be reduced to categorical ones. And what he says about categorical judgements might lead one to the conclusion that – as far as conditionals are concerned – the differences between Philo and Chrysippos, and Frege and Lewis, shouldn't bother us more than the simple fact that a word like *everybody* may sometimes be used for referring to a small group of people and at other times for talking about mankind as a whole.

2. Like Chrysippos, John Wallis believed that only those conditionals are

true which have the right kind of connection between their antecedent and consequent. Referring to conditionals and other compound propositions (*propositiones compositae*), he writes:²

Veraeque aut falsae censendae sunt prout connexio illa rite aut perperam facta est; quaecumque fuerit veritas aut falsitas, partium separatim consideratarum.

(They have to be judged true or false according to whether that connection is correctly or falsely made, whatever the truth or falsehood of their constituents may be if considered in isolation.)

In this definition, there is one word which is particularly vague and difficult to understand: *Connexio* (*connection*).

In stepwise fashion, Wallis tries to get more explicit about it. A conditional that he takes to be true is expressed by the following sentence:

Si Sol oritur, dies est.
(If the sun rises, it is day.)

We are told that in this conditional, there is the right connection between antecedent and consequent. A conditional that lacks this connection is expressed by the following example:

Si Alexander sit Homo, tum Bucephalus est Equus.
(If Alexander is a man, then Bucephalus is a horse.)

Although antecedent and consequent of this conditional are both true, it is still false according to John Wallis:

Non enim inde sequitur Bucephalum esse Equum, si Alexander sit Homo.
(The reason is that it does not follow that Bucephalus is a horse if Alexander is a man.)

Here, John Wallis shifted whatever was blurred in the meaning of *Connexio* on to another word: *sequitur* (*follows*). For Wallis, the word *sequitur* doesn't mean the same as *follows logically*. Whenever he wants to talk about logical consequence – for example when he discusses the validity of inferences – he puts the adverb *necessario* (*necessarily*) in front of the verb *sequitur*.

We find *sequitur* with still other adverbs in the *Institutio Logicae*. For example, when causal dependence is discussed. For Wallis, a causal dependence between two propositions is based on a relation of natural or essential consequence between those two propositions, that is on a relation of *sequitur naturaliter* or *sequitur essentialiter*. He illustrates this with the following sentence:

Quia Sol oritur, dies est.

(Since the sun rises, it is day.)

The proposition expressed by this sentence is true if and only if it follows 'naturally' or 'essentially' that it is day from the fact that the sun rises. Wallis considers this causal relation to be something 'more' than the pure consequence relation (*nuda consequentia*) which is required for conditionals. What this eventually amounts to will never become quite clear. Wallis however undertakes further efforts to elucidate the issue. He asks himself: What is the denial of a conditional? What is the denial of the proposition expressed by the following sentence?

Alexander, si fortis sit, vincet.

(If Alexander is brave, he will be victorious.)

The answer is: A conditional is denied by denying the connection (*copulam negando*), that is by denying the consequence relation between antecedent and consequent (*Consequentis ab Antecedente sequelam*). The denial of the above sentence should then be equivalent to

Non sequitur enim, eum certo victurum esse, si fortis sit.

(It does not follow that he is certain to be victorious if he is brave.)

This sentence is in turn equivalent to

Fieri potest, ut non vincet, etiamsi fortis sit.

(It is possible that he will not be victorious, even if he is brave.)

This suggests that John Wallis considers conditionals to be implicitly modalized. In possible worlds terminology, the proposition expressed by the above sentence would be true in a world if and only if there is an (accessible) possible world such that Alexander is brave but yet won't be victorious. This is actually quite close to Wallis' final step in the reduction. For the undenied 'Alexander' – sentence, this reduction would look as follows:

Every case where Alexander is brave is a case where he will be victorious.

Wallis' original example is:

Si sol splendet, dies est.

(If the sun shines, it is day.)

which is reduced to the categorical statement:

Omnis casus quo sol splendet est casus quo dies est.

(Every case where the sun shines is a case where it is day.)

There is still a lot of vagueness in this reduction. What are 'all' cases? All imaginable cases or all cases of a certain kind? Let us consider now what John Wallis himself says about the interpretation of quantifiers like *all*, *every*, *none*, etc.:³

Et quidem ipsae Universalitatis (utcumque Hyperbolice prolatae) saepenumero nihil aliud significant, quam quos *Universalibus* supparet modo diximus. Utoputa cum in communi colloquio quotidie dicitur, *Hoc omnibus notum est, Nemo hoc ignorat, Nemo inficiabitur*, etc. (Gallis familiare illud) *Tout le monde*, (ubi forte tres aut decem sunt ille totus mundus), *Nil tale unquam contigit, In hoc conveniunt Omnes*, non aliud intellegendum erit quam *Plerique* aut *Paucissimi*.

This means roughly the following:

In colloquial speech, words like *all*, *every* or *whole* do not always express real universality but only quasi-universality. Wallis gives the French example *tout le monde* (*tout le monde* in the original). This expression may sometimes refer to only three or ten persons which constitute the whole world in this particular context. We find here the roots of what was baptized a *universe of discourse* by George Boole. The following passage is taken from the *Laws of Thought*:⁴

In every discourse, whether of the mind conversing with its own thoughts, or of the individual in his intercourse with others, there is an assumed or expressed limit within which the subjects of its operation are confined. The most unfettered discourse is that in which the words we use are understood in the widest possible application, and for them the limits of discourse are coextensive with those of the universe itself. But more usually we confine ourselves to a less spacious field. Sometimes, in discoursing of men we imply (without expressing the limitation) that it is of men only under certain circumstances and conditions that we speak, as of civilized men, or of men in the vigour of life, or of men under some other condition or relation. Now, whatever may be the extent of the field within which all the objects of our discourse are found, that field may properly be termed the universe of discourse.

A lot of contemporary work in semantics deals with the fact that the interpretation of many expressions of natural language depend on a parameter that has to be fixed by the utterance context. It is part of our knowledge of the meaning of these expressions that we know which parameters have to be fixed: the speaker for *me*, the listener for *you*, the place of the utterance for *here* and so on. Very often in our conversations, these parameters aren't fixed in a determinate way. As a result, what we say is vague.

What Wallis and Boole observed, is that quantifiers are context-dependent expressions, too, and that the relevant parameter is a universe of discourse.

Given Wallis' analysis of conditionals, this means that they should be context-dependent in much the same way as quantifiers are and that their interpretation should depend on a universe of 'cases' which may vary from situation to situation. The difference between Philo and Frege on one side and between Chrysippos and Lewis on the other side may then be seen not as a difference in interpreting conditionals but rather as a difference in fixing the open parameter.

In modern terms, Wallis' analysis amounts roughly to taking conditionals to be variably modalized material implications. The logical form of *if . . . then* clauses would then be as follows:

Necessarily ($A \supset C$)

The proposition expressed by an utterance of a sentence of this form would be true in the actual world if and only if the proposition expressed by C is true in all those 'accessible' worlds in which the proposition expressed by A is true. If all possible worlds are accessible from the actual world, then the corresponding conditional is strict implication. In the degenerate case (rejected by Wallis) where no other world but itself is accessible to the actual world, the resulting conditional is material implication. And there are all sorts of implications inbetween – possible candidates for providing a more or less strong connection between antecedent and consequent.

We may wonder whether this analysis of conditionals is plausible according to modern standards. In the following section, I'll give a short illustration of the kinds of objections it would meet today.

3. There is still quite a number of people defending material implication as the right interpretation for indicative *if . . . then* clauses. For them, no context-dependency affects their truth-conditions and their logical form is simply

$A \supset C$.⁵

No implicit modalization has to account for a connection between antecedent and consequent. That sentences like Frege's 'cock'-sentence and Wallis' 'Bucephalus'-sentence sound strange is attributed to independently motivated pragmatic principles. It would simply be inappropriate to utter these sentences, although what is said by uttering them is true. There is some discussion about what these independently motivated principles are. One might be

sceptical about them as Lewis is in [12]. My own qualms, however, concern a different point: There are some types of *if . . . then* -clauses (including indicative ones) that can't possibly be formalized by means of material implication.

Consider the following sentences:

- (1) If John has stolen the earrings, he must go to jail (in view of what the law provides).
- (2) If John has stolen the earrings, he has probably sold them to a friend.
- (3) If John had stolen the earrings, the dog wouldn't have barked.

Formalizing sentences (1) to (3) as

$A \supset (\textit{necessarily } C)$

$A \supset (\textit{probably } C)$

$A \supset (\textit{would } C)$

respectively would lead to the absurdity that the corresponding conditionals would all be true just in case John hasn't stolen the earrings. I can't see any pragmatic principles that could save the analysis.

A formalization à la Wallis doesn't fare much better. We would take the modals to have wide scope over the whole sentence, that is we would have something like the following:

$$\left. \begin{array}{l} \textit{Necessarily} \\ \textit{Probably} \\ \textit{Would} \end{array} \right\} (A \supset C)$$

We would assume that, accidentally, the modals crop up in the consequent, if conditionals are explicitly modalized. And to these modals we would give an interpretation roughly along the following lines:

The proposition expressed by an utterance of a sentence of the form

(in view of what the Law provides) necessarily A

is true in a world *w* if and only if the proposition expressed by *A* is true in all worlds compatible with what the Law provides in *w*.

The proposition expressed by an utterance of a sentence of the form

probably A

is true in a world *w* if and only if the proposition expressed by

A is true in all those worlds that are compatible with what is considered to be the normal course of events in *w*.

The proposition expressed by an utterance of a sentence of the form

would A

is true in a world *w* if and only if the proposition expressed by *A* is true in all those worlds which are compatible with a certain set of 'relevant' facts of *w*.

These suggestions lead to many problems.

As for sentence (1), interpreting it as a modalized material implication, leads to the well-known paradoxes of deontic logic.⁶ If John breaks the Law in stealing the earrings, then the analysis predicts that the conditional expressed by (1) should be true vacuously.

Similar paradoxes arise for (2), in case John's stealing the earrings is not compatible with what we consider to be the normal course of events.⁷ And as for (3), there is the difficulty of being more specific about these 'relevant' facts. This is basically Goodman's problem of cotenable extra premises which he had to leave unsolved.⁸

There is another difficulty concerning all three types of sentences: For modalized material implications, the three inference patterns of 'strengthening the antecedent', 'transitivity' and 'contraposition' are predicted to hold. Yet it has often been pointed out that these inference patterns are not valid for deontic conditionals, probability conditionals and counterfactuals.⁹ As an illustration of 'strengthening the antecedent' consider the following examples:

- (1') If John has stolen the earrings and has shot himself straightaway, he must go to jail (in view of what the Law provides).
- (2') If John has stolen the earrings and has destroyed them straightaway, he has probably sold them to a friend.
- (3') If John had stolen the earrings and had teased the dog, the dog wouldn't have barked.

Given that the propositions expressed by (1), (2) or (3) are true we can't infer that the propositions expressed by (1'), (2') and (3') respectively are true as well.

Now this means that anyone who insists on interpreting 'ordinary' indicative *if . . . then*-clauses in terms of modalized (or bare) material implications, would have to concede that sentences like (1), (2) or (3) would have to be interpreted in a completely different fashion. This is an undesirable

conclusion, since *if . . . then* doesn't seem to be ambiguous. Furthermore, there is still a non-trivial common core of inference patterns that are valid for both types of conditionals.¹⁶ Here is an example:

- | | | |
|---|---|--|
| | If <i>A</i> , then <i>B</i> | |
| | If <i>A</i> , then <i>C</i> | |
| | | |
| ∴ | If <i>A</i> and <i>B</i> , then <i>C</i> | |
| | If <i>A</i> is the case, then <i>B</i> ought to be the case | |
| | If <i>A</i> is the case, then <i>C</i> ought to be the case | |
| | | |
| ∴ | If <i>A</i> and <i>B</i> are the case, then <i>C</i> ought to be the case | |
| | If <i>A</i> , then probably <i>B</i> | |
| | If <i>A</i> , then probably <i>C</i> | |
| | | |
| ∴ | If <i>A</i> and <i>B</i> , then probably <i>C</i> | |
| | If <i>A</i> were to be the case, then <i>B</i> would be the case | |
| | If <i>A</i> were to be the case, then <i>C</i> would be the case | |
| | | |
| ∴ | If <i>A</i> and <i>B</i> were to be the case, then <i>C</i> would be the case | |

I think that these considerations suggest that we should look for a uniform analysis of conditionals that is flexible enough to allow for differences, but that is also restrictive enough to characterize the common core. This is what I attempted in [10] and some earlier papers. The analysis given there is indebted to Wallis for stressing context-dependency and the intrinsic connection between modals and conditionals. And I think it saves enough of his insights to let the four old men sleep in peace.

NOTES

* This paper is a radical revision of an earlier version that I wrote while being a researcher in the Sonderforschungsbereich 99 at the University of Constance, sponsored by the German Science Foundation.

¹ Frege [5], page 84. The English translation is taken from R. Stoothoff (with minor changes), *Mind* 62, 1963.

² This quote as well as the following ones (until further notice) is from Wallis [13], pages 139ff. The translation is my own.

³ Wallis [13], page 126.

⁴ Boole [2], page 42.

⁵ See Grice [7] or Jackson [9].

⁶ See Hanson [8], van Fraassen [4].

⁷ See also Lewis' triviality results in [12] for an objection of a more general kind.

⁸ Goodman [6].

⁹ See Lewis [11], Adams [1].

¹⁰ See Burgess [3].

BIBLIOGRAPHY

- [1] Adams, E. A.: 1975, *The Logic of Conditionals*, Dordrecht, Reidel.
- [2] Boole, G.: *An Investigation of the Laws of Thought*, New York, Dover Publications, reprint of the 1854 edition.
- [3] Burgess, J.: 1979, *Quick Completeness Proofs for some Logics of Conditionals*, Manuscript, Princeton University.
- [4] van Fraassen, B.: 1972, 'The logic of conditional obligation', *Journal of Philosophical Logic* 1, 417–438.
- [5] Frege, G.: 1966, *Logische Untersuchungen*, G. Patzig (ed.), Göttingen, Vandenhoeck and Rupprecht.
- [6] Goodman, N.: 1947, 'The problem of counterfactual conditionals', *The Journal of Philosophy* 44.
- [7] Grice, H. P.: *The William James Lectures*, unpublished.
- [8] Hanson, B.: 1969, 'An analysis of some deontic logics', *Nôus* 3.
- [9] Jackson, F.: 1979, 'On assertion and indicative conditionals', *The Philosophical Review* 88.
- [10] Kratzer, A.: 1981, 'The notional category of modality', in H. J. Eikmeyer and H. Rieser (eds.): *Words, Worlds and Contexts. New Approaches in Word Semantics*, Berlin–New York, de Gruyter.
- [11] Lewis, D.: 1973, *Counterfactuals*, Oxford, Blackwell.
- [12] Lewis, D.: 1976, 'Probabilities of conditionals and conditional probability', *The Philosophical Review* 85.
- [13] Wallis, J.: 1699, *Opera Quaedam Miscellanea*, Oxford. Reprinted in Wallis, J., *Opera Mathematica*, Vol. III, Hildesheim and New York, Olms, 1972.