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If all our arguments had to be conclusive, there would be few of them

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IF ALL OUR ARGUMENTS HAD TO BE CONCLUSIVE, THERE WOULD BE FEW OF THEM

And if they were, of us.[1] But we are plentiful, and so are they. Given this fact of current life, a theory of meaning whose backbone is proof positive[2] cannot be general. Since proof is a key ingredient of the most well-developed computational theories of mind, objections to it are also objections to the most straightforward, procedural explication of individually psychologistic Conceptual Role Semantics.[3]

Proof-conditional theories of meaning are set off from essentially and avowedly truth-based ones by the insistence that truth-conditions unverifiable in principle or practice are not a sound basis for a theory of meaning.[4] Was there a penguin within five feet of the south pole thirty days ago? Questions of this kind, and others involving the distant future, have been offered as counterexamples to truth-conditional foundations for a theory of meaning. But the range of application of a proof-theoretic semantics will be circumscribed no less severely than that of its truth-conditional foil. Everyday means for getting an answer to the question concerning the penguin are as close to hand as those for knowing whether or not someone's software did give us a proof of the Four Colour Theorem. To argue, then, that proof positive informs everyday linguistic practice — 'grasp' rather than proto-juridical imputability — involves a leap of faith comparable to that which antirealists impute to their opposite numbers.

[1] No joke. The first thing that enters a mind concentrated by sudden anger or reckless greed often enough speaks for the radical approach to significant others.

[2] Cf. Dummett (1976), Martin-Löf (1984).

[3] Embodied in the classic showpieces of artificially intelligent language understanding.

[4] I use the term interchangeably with 'meaning theory'. Let context decide whether reference is to a theory imputable to natives, a theory of such a theory espoused by the analyst, or a theory about the possible form one or the other of the preceding might take.

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There are, of course, other objections to the viability of a verificationist theory of meaning, not least because it is far from clear what it would amount to once expressions of propositional attitude are being considered. But then, attitude contexts (Kim believing that Sandy talks) are problematic for any would-be compositional theory of meaning, as Schiffer (1987) has argued.[5] In what follows I shall try to make a substantive case for a theory of meaning based on the specific objection levelled at the proof-theoretic approach.

Section 1 states a few methodological assumptions.

Section 2 argues in effect that treating semantics as a social phenomenon is to treat it as a political phenomenon, and that one ought not pretend otherwise by ignoring political process.

Section 3 takes the individualistic perspective and a standard line on the fact that people speak idiolects.

Section 4 takes up the title, considers the Aristotelian (and earlier) notion of enthymeme, asks what can be done with it in formal terms, and takes a decision-theoretic line on jumping to conclusions.

Section 5 considers non-conclusive argumentative relations and gives a standard definition of relevance, with a side-remark on Tractarian Wittgenstein.

Section 6 proves some results about non-deductive, argumentative characteristics of logical connectives. One of them, negation, is fully characterized in terms of the relations 'for' and 'against'. Conditions for relevance additivity of conjunction and convexity of disjunction are given (leading to an argument about apparent exclusivity of 'or'), and two hypotheses on the structure of human categorization stated and supported. Some familiar relevance functions–i.e. numerical representations of decision-theoretically coherent epistemic context-change potential—are shown to be cardinal utility functions.

Section 7 explores a fairly radical step. Taking its cue from the results on additivity and convexity, it asks whether a semantics could be induced directly by utility or relevance valuations, just as Boolean semantis is by truth valuations. A semantics in linear algebras over the real numbers[6] is given for coordination, which makes connectives indexical. A

[5] This is a matter I shall not attend to here. My feeling is that one should either discuss it at the level of technicality and explicitness exemplified in Montague (1974), or leave it.

[6] Like elementary probability theory, linear algebra is part of the curriculum of any western European high school leaver, never mind first year science student. In the unlikely event of a certain feeling arising, namely that one knows well enough what meaning intuitions could possibly be about to also know that one wouldn't wish to be bothered with this kind of mathematical structure, it is worth remembering the door sign at the Academy and suppressing the feeling. One might, instead, try to find

little-known problem for Boolean and other lattice-theoretic semantics in plain assertoric indicatives is used to illustrate descriptive advantages. Brief remarks on how to get back to truth-conditions are added.

1. Methodological Preliminaries

Philosophy, which, like any discipline, is defined most clearly by its presuppositions, sits uneasily between psychology and jurisprudence. In the cognitive sciences its ambiguous position between the descriptive and the prescriptive has been apparent at the very least since Boole (1854), who opted firmly for a normative interpretation of the Laws of Thought.[7]

There is nothing objectionable about this if, like Frege and, most probably, Boole, we are in quest of an ideal language of communicable thought subject to ideal conventions of inference. But if a theory of meaning (the analyst's) is to be that for an extant natural language or a plurality of such languages, vacillation between the two enterprises—a habit whose legitimacy is taken for granted in most philosophy of language—is no longer so unproblematic.

Take a look at those sciences which, by all pertinent criteria, have a claim to the name[8]. The impression must be that progress within them has come about by a mixture of ingredients whose least negligible components are detailed observation (with experimentation the special case thereof) and mathematization (with a heuristic premium on elegance of structure). Advances in posing transcendental questions in something like the sense of Kant—i.e. in exploring the conditions of possibility for the non-emptiness of certain traditional theoretical concepts of philosophy—have been driven by such advances.[9]

To propose a descriptive theory of meaning is, virtually by definition of the moral sciences, to propose — either exclusively or as a prominent first step — a description of norms governing ordinary, pre-philosophical conduct. That such norms may be violated is commonplace; but the analyst's data are, not least, typical reactions to such violations, which attest to the pertinence of such norms for the way our minds work (together). There may be all sorts of levels of norms in operation at the same time,

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out where, if anywhere,the exposition commits mathematical mistakes or empirical infelicities.

[7]Chapter XXI, cf. e.g. p. 408 on the 'laws of *right* reasoning' and their 'actual transgression'.

[8]I.e. those which philosophers of science published in the eponymous journal by and large deem worthy of attention.

[9]If anyone would tell me that argumentation about the possibility of action at a distance is idle talk these days unless informed by an understanding of Bell's Inequality (i.e. a large chunk of physics), I should immediately believe them.

as anywhere else in social process. Think of a public official who operates by the norms of te Civil Service Guidelines on Conduct, but then, while doing his best to preserve appearances, takes a kickback. Of course, the latter set of activities has its own set of norms (e.g. to deliver in return, preserve confidentiality, and so on).

Returning now to less introspectible norms (that might well shade into intransgressible constraints on behaviour), their investigation should not be *a priori* a task less exacting than to propose a theory of rigid body kinematics, fluid dynamics or wave mechanics. To assume the contrary is to assume either that operations of the relevant parts of mind are very much simpler than, say, the workings of a spinning top[10], or that semantics is, after all, not an empirical discipline and therefore not bound to explanatory standards set by the most successful of such disciplines.

By these standards, theories of meaning for natural languages are still somewhat patchy affairs. Anyone proposing to make a contribution to the genre must then choose a patch to dig, sow, and maybe reap. In the empirical language sciences, it was Montague's truth-conditional attempt to account for native speaker intuitions on acceptability, paraphrase and, as a generalization of the latter, consequence that captured the imagination.[11] By immediate contrast, Davidson's programmatic and comparatively sketchy approach that also assigned truth a central role has been talked about almost exclusively within philosophy.[12] In return, there is many a philosophical book on semantics, including such things as propositional attitude sentences, which does not as much as mention Montague though written decades after his version of truth-conditional semantics (TCS) became an instrument of explanation.

What is the point of pontificating? This. I shall take the leading observation as my point of departure. I will then ask whether a compositional account of native speaker intuitions for a part of English language structure (wrongly) deemed unproblematic in most of current philosophy and linguistics—roughly, the analogue of propositional or first-order classical logic—can be built on a view of communication motivating the observation.

[10] Sommerfeld's monograph on this piece of child's play runs to around a thousand pages of mathematical physics.

[11] As operationalized in person-hours and research funds spent on working out its consequences or improving it by more or less radical critique.

[12] To be sure, the device of quantifying over events, first proposed by Ramsey (1927), which Davidson applied to the analysis of action sentences, has been widely adopted in 'flat'-structure post-Montagovian semantic approaches. But adoption has been for technical convenience, not as an endorsement of Davidson's general philosophy of meaning.

The means to be employed are familiar from decision theory and mathematical psychology. After some intuitive and historical considerations I will first pose a question about grasp of standard boolean connectives in terms of argumentative relations short of proof positive. Some results will be derived, characterizing one of these connectives (negation) and suggesting certain constraints on assumptions underlying the use of two others (disjunction, conjunction). I shall then ask to what extent speaker intuitions conforming to these constraints will be predicted by a framework for algebraic recursion which takes its ultimate semantic values in utilities rather than truth or similar proof-game success values.

The hope is to indicate, by means of a narrow case study, how a description with some fine structure and perhaps some aspirations to elegance predicts data problematic for the currently received family of descriptively committed theories. A further hope is that it will also come to serve as a heuristic for a fresh approach to matters that *are* by now widely deemed problematic.

The investigation will seem cavalier in doing little to answer scruples motivating recognition of these problematic features of current theories as problematic. (Some hints on attitude contexts are given, but I am not sure how much they help.) Remember, though, that philosophy appeals to intuitions. If it aspires to being a science, it cannot afford not to ask: Where do these intuitions come from? My first objective, accordingly, is to get a better hold on the philosopher's explicandum.

The intuitions to be accounted for are not of a rare kind. We should not appeal to them in deciding whether or not the inhabitants of Twin Earth are indeed referring to Water when talking of a perceptually indistinguishable substance that is not H_2O. Such questions are not settled by experiment: the Supreme Court will deal with them when the time comes, and (the way Putnam's set-up is constructed) without deference to expert witnesses.

Having situated the inquiry within the cognitive sciences let me now propose a substantive thesis. It starts out eminently philosophical; but the objective I shall try to attain is to turn it into a claim about the psychology and anthropology of language that can be debated and maybe settled by appeal to experimental data (quick and dirty native speaker judgments) and appeal to considerations of simplicity. (Simplicity all things considered: ancillaries included.)

The leading idea is to interpret the notion of 'use', appropriated by champions of proof-conditional semantics (PCS), in the general, everyday pragmatic sense of 'utility': distinct from 'usage' (its dominant reading in later Wittgenstein) and much more general even than the very special activity of mathematical proof.

There have, of course, been suggestions to define word meaning in terms of the word's contribution to the 'illocutionary potential' of sentences in which it occurs. Such potentials might be diverse enough to cover any conceivable interpretation of use. But without a definition of use specific enough to generate recursive structure (as 'truth' and 'proof' do) the suggestion remains a *flatus voci*. Any of the currently traded symbioses of TCS or (PCS) and Austin/Grice ancillaries of speech action and implicature would fit the bill.

An approach to meaning that takes on board a prima facie commitment to some form of compositionality or other[13] is like a country without an army or a navy if it does not command algebraic structure of its own. 'Its own' means, something distinctive of its values or, if you will, its way of answering the question: 'Why talk?' TCS and PCS each have their own distinctive brand of algebra: Boolean in the first case, Heyting in the second. If they gave no trouble, even before we consider propositional attitudes, the quest for a native fighting or sales force would be extravagant. But, as we shall see, they do give trouble which, to the best of my current knowledge, no conservative remedy is available for.

What, then, might an algebra of use-conditional semantics (UCS) look like if use is explicated as a form of utility? A cognitive scientist, someone without settled views on the meaning of meaning, might be willing to investigate all sorts of algebras. In the final section I shall propose one: nothing handstitched, but well known and found all over the natural, social and cognitive sciences.[14]

The prerequistes assumed in what follows will be no more than a relatively small proper subset of those needed to make meaningful statements about neural networks: some probability theory, some linear algebra. No commitment to such network representations is presupposed. However, the approach to meaning I shall pursue is perhaps more palatable to those who are happy with the idea that neural networks underlie our cognitive capacities than to those who find their lack of introspectible accessibility unsettling.

2. Meaning as a Matter of Political Influence

[13] Combinatorial creativity is a fairly brute fact of linguistic life that won't go away even if our current theories of language learning were to.

[14] As Dick Oehrle of the University of Arizona reported someone as saying: linear algebra is the kind of algebra where everything works.

The currently received working linguist's view of meaning is predicated on the idea of information transmission.[15] Consistent with priority of attention granted to the indicative mood, the ideal communicative situation underlying the ruling symbiosis of broadly truth-conditional Tarskian semantics and Gricean pragmatics is a game of pure cooperation (a.k.a. team game) as expounded in Lewis (1969) to foundational ends.

Information is transmitted either disinterestedly or to optimize benefits in a situation where participants' utilities are increasing functions of one another's. Analysis of imperatives then takes the form of a Master-Slave situation (explicitly so in Lewis 1970) in which the question of naively autonomous preferences never arises for Slave.[16] Of course, philosophers of meaning—indeed Grice himself—were fully aware that partisan dispute is a prominent linguistic activity. But if one looks at the explanatory structure of the doctrine these matters turn out to be arising after office hours.[17]

The ideology of communication I shall try to milk for empirical structure is basically interested and partisan. Its paradigmatic context of use is constituted by two persons that have to negotiate a joint strategy, i.e. a mutually binding constraint on their conduct (typical imperative case) or disposition to conduct (typical indicative case on the dispositional construal of belief). If they were of one mind—in precisely the sense of the idiom that designates consonance of preferences—there would be no issue between them. Issues live on locally inverse preferences, and on the provision of incentives for one or more of the participants to accomodate to the other's preferences. Incentives might range from wielding a heavy stick (the original *argumentum ad baculum*), over offers of money or charitable status, and (in the typical indicative case) tacit or explicit offers of whatever is common evidential currency.

[15] So is that of the philosopher committed to intention-based semantics: if smoke means fire, it transmits information to an observer; if Sandy means something by showing Kim a picture of Lee and Kim's lover on the beach, Sandy, or Sandy's gesture transmits information to Kim; and so on towards sentence- and word-meaning. Whatever causal relations there are between sign or utterance on the one hand and comprehension on the other, they factor through information transmission.

[16] Consistent with this indicative bias Lewis avails himself of the explicit performative format to argue that Master's pronouncements automatically bring about the truth of the corresponding obligation statements.

[17] The possibility of deception is, no doubt, addressed in the insistence on truthfulness and evidential backing. But it is not distinguished in that from plain error: it adds no structure, as for instance an assumption of opposed preferences would immediately. By contrast, opposed preferences do generalize nicely if all we want in a given moment is a safeguard against inadvertent error.

The earliest statement of this general view of human communication in modern philosophy is due to Mandeville (1729). People speak so that their thoughts may be known, says the simpleton, chock-full of common sense. No, replies Mandeville's mouthpiece, they speak to get their way, and have done so for all of speaking time. Coming from the originator of the notion 'division of labour' and, more famously, of the 'individual vices, collective benefits' argument, this line of thinking looks dangerously prone to serve as an apology for the 'natural' justice of Thatcherism, Reaganomics and similar euphemisms for getting the fewer rich quicker.. But such all-too conceivable dereliction of scientific duty should not, in itself, stand in the way of linguistic description. The object, precisely to the extent of informing general semantic interpretation, might be a fairly low-level interpretive convention, preserved in virtue of generating useful recursive structure rather than in deference to and support of a deeply felt Weltanschauung.

A default assumption of opposite preferences (opposite on the Pareto boundary of a mixed-motive bargaining situation, rather than in the empirically rare sense of a game of strict competition) is no less a candidate for a default than the received assumption that preferences are consonant enough to be ignored altogether. Neither assumption will always hold in they eye of human or divine observers. Which default delivers structure is entirely an empirical question.[18]

In what follows I shall concentrate on the indicative case and will treat evidential relevance as explicated within the probability calculus as a utility.[19] The basic idea is that propositions and their algebraic compounds are evaluated by (positive or negative) relevance with respect to a dichotomous issue.

Put crudely, the gross value of an imperative to the utterer is the utility increment to be derived from the addressee's compliance. If values

[18]Of course, the utter silence on matters political that is currently observed by non-peripheral contributions to the philosophy of language, meaning, and mind will add a sociological sheen to the matter. And its happy consonance with a profitable ideology of social control—let the fictional enterprise Diznosoft stand for it—adds an evaluative gloss.

[19]Good (1983) has called the particular form he champions 'weight of evidence', noting that it is what he calls a 'quasi-utility'. Below I shall point out that it and a related function is in fact a cardinal utility.

are deemed intersubjective[20], we shall, on disregarding incentives offered, have a simple transfer of standardized utility before us.

"Give me your wallet!" is not a bad example sentence. "Give me your wallet and your mobile phone", "Give me you wallet or your mobile phone" are others. These utterances are, presumably, of the speech act type "demand" or "claim" (demanding with menaces being a special case). Whereas a response chosen from among "OK, take my wallet ({and/or}) my mobile phone)" would presumably instantiate a concession.

Already here we are ready for an immediate explanatory payoff whose value will be an increasing function of the time one has spent pondering the underlying problem. Note that a demand (Nash 1953) is bounded from below[21]: if I ask for 5 dollars, I should not rationally mind if you give me 6, 1000, or your Mercedes into the bargain. But I shall get very edgy, indeed angry, starting with 4.99 and going down. Dually, if I concede 5 dollars to you, I don't mind if you take less (or nothing, or give me a dollar or more) but I will mind if you give me more.[22]

Consider now disjunction and conjunction. For demands (homogeneous in sign of utility) an inclusive reading is predicted for disjunction; likewise, conjunction has its ordinary reading. For concessions we should predict that disjunction will have an exclusive bias: the least concession compatible with one disjunct being conceded is the natural reading. Similarly, if I concede the conjunction you are free to take just one or even neither. Suppose commands are being represented as a species of demand, and permissions as a species of concession. Then the above informally stated predictions will presage a difference between readings for these two act-modes which correspond precisely to those not predicted by current context-change descriptions of the phenomena.[23]

Boolean and weaker lattice-theoretic accounts of coordinating connectives, even when aided by Gricean pragmatics, are unable to predict correctly under the context-change model of permission proposed by Lewis

[20] This is a naive assumption underlying Aristotle's account of economic exchange, discredited by 19th century marginal utility theory, and beset by all the philosophical and mathematical difficulties attaching to the very idea of interpersonal utility.

[21] And from above only by the feasible set, whose extent may be imperfectly known to players under incomplete information.

[22] We take for granted naive independence and monotonicity assumptions that reflect in surprise at the aesthetic fact that milk and lemon jointly don't improve your tea, or in literary works such as the Sorcerer's Apprentice.

[23] If interested in the non-homogeneous case, read my 270 pp. 1996 Habilitationsschrift on the semantics of 'but'; and reconstruct the explanation for why the occurrence of the first conjunct in 'Say another word and I'll blow your head off' is negative, in a preferential sense nicely captured in Siegel/Eastwood's 'Make my day' episode.

(1970) and tested with respect to disjunction by Kamp (1979); cf. Merin (1992) for the demonstration. The 'free choice permission problem', conceived of as one of non-monotonic theory change in the approach of Lewis, Kamp and (by private communication to Lewis) Stalnaker is a test case for current semantics: in its imperative version well before we encounter modal and propositional attitude verbs. Everything goes smoothly for commands; everything goes haywire for permissions and—for the interesting case of compound sentences—remains recalcitrant to those remedies (nested sets of ever more impermissible possible worlds) that spell success for the atomic case once we grant the existence of an appropriate similarity relation on worlds.

Intuitions of analogous differentials are less clear in the case of indicatives, at least in the case of conjunction. The corresponding difference between assertion and admission is still intelligible: my assertion is something whose concession by the opposing party is prima facie good for the case I am pursuing, my admission what is prima facie bad for it. Nonetheless, to the extent that evidential incentives are impersonally given (supplied by Nature) there is only limited room for manoeuvre. The situation most conducive to intuiting a differential is one where the difference makes a difference: i.e. judicial process where it is frequently up to the discretion of a party which of their opponent's admissions they might wish to have entered into the record or final reckoning.

The assumption, then, is that semantics for natural languages is part of political economy and that the role of commands, demands, assertions, denials and the like is the exercise of power. Power, says Max Weber (1922:Par.16), is the chance of being able to let one's will prevail within a social relationship, even against resistance. Harsanyi (1962) went on to explicate this idea in terms of raising the probability of another's actor-desired action within the bargaining game framework of Nash (1953).

If it is indeed probabilities of action (I do not exclude mental action, assuming we can attach a sense to the notion of mental act) that assertions and the like are aimed at transforming, then a probabilistic view of conceptual role becomes much less implausible than under a view of meaning modelled on the language game of judicial testimony. It might seem plausible that someone knows whether or not he believes that Smith hit Jones; it might seem rather less plausible to expect introspective knowledge of one's precise or even approximate degree of belief.

However, if our role model is not the witness of forensic discourse, nor the terminal appendage patiently awaiting its latest update from Diznosoft, then avowals and blind instant faith give way to changes of more viscous disposition. And then it is no longer so implausible to impute individuals with 'probability theories' in the sense of Ramsey (1929), i.e. sets of subjective conditional probability assignments, as representations of their beliefs about the world.

3. Conceptual Role and the Problem of Idiolect

Inductive relations that make up the meat of most proposals for conceptual role semantics (CRS) are notoriously prone to wiping out the distinction between linguistic (lexical) knowledge and world knowledge in general. The paradigm case of linguistic knowledge is given by entailments of the form $A \wedge B \vdash A$; CRS characteristically involves inductive relations to propositions C not identical to Boolean functions of A and B. Let me for mnemonic convenience call the former kind of relations *endocentric* and the latter *exocentric*.

Inductive relations also raise, more acutely perhaps than less fine-grained theories, the problem of inter-speaker communication: it seems just too implausible to assume that two persons at any given moment have the same subjective probability function.

If evidential relevance is to be our mainstay for a semantics of simple indicatives, we are almost inexorably committed to some form of conceptual role semantics of the kind outlined by Field. Its condition for two sentences, A, B to have the same conceptual role is relative to an epistemic state represented by a probability function and is phrased thus: $\forall C : P(A|C) = P(B|C)$. No matter what C you assume, your degrees of belief in A and in B will always remain identical.

The interpersonal problem (of which the problem of intra-personal identity over time generates a variant) is one I cannot hope to solve in its generality. On the other hand, if language understanding is our problem, then, in the absence of divine intervention, the problem of how idiolects blend into dialects and languages is not proper to the probabilistic framework. Even non-compositional accounts of animal communication will have to come to terms with the fact that no two opposite-sex exemplars of a sexually dimorphic species are wholly identical: and yet they manage to be fruitful and multiply.

The interim solution or way around the problem I shall adopt is due to Rohit Parikh (1994). He points out that communication may be successful in terms of nicely operationalized criteria even when there is no assumption that individual's extensions for predicates such as, for instance, 'blue' or 'green' are identical. Provided there is sufficient overlap between your extension for 'blue' and mine, your telling me 'It has a blue cover' will save me an appreciable amount of search time when I go off to fetch your copy of 'Naturalistic Epistemology'. (Your books are not ordered by author, title or subject.)

My proposal, in line with Parikh, is to sideline worries about intersubjectivity. I shall assume that—as a matter of fact, explicable in biological-cum-sociological terms—individuals making up a language community are constructed and socialized in a manner sufficiently uniform for their notions to have sufficient overlap of extension so as to make the differences negligible for most intents and purposes. Propositions or Fregean Thoughts, one of the mainstays of current formal analysis of meaning, would then be convenient metapragmatic or language-scientific theoretical constructs which one might wish to eliminate ultimately in favour of something as well-understood as causal relations.[24] What makes Parikh's proposal rather more than a useful stopgap for present purposes is its truly pragmatic nature: utility and probability—here not least objective probability–play an explanatory role in the account. (Something unobvious is being added.)

4. Enthymeme

Let us return to the initial charge. I should uphold it even while taking for granted that appeal to use *qua* proof does shed much light on those human activities whose evident subject matter is broadly mathematical. Such activities are, no doubt, highly and monetarily consequential language games — as I shall be happy to call them. But they are not the games most people play much of the time.

Nor could they be. Arguments on matters of fact in a Humean world can but incline without necessitating.[25] One can be swayed by an argument—sufficiently so to some particular intent and purpose—without being knocked down even for a moment let alone all time, intents and purposes. Similar things might well be said about another domain of discourse where even the criteria of argumentation themselves apear wide open to debate: disputes on what ought to be, of which a special case is what counts as proof.[26]

[24] Sure, this *is* a joke: the moral might well turn out to be that, if the second is as ill-understood or aporetic as the first when you really put your mind to it, you might as well keep the one that's convenient to reckon with.

[25] Leibniz's happy phrase, adapted to mundane ends.

[26] Recall again the Four Colour Theorem, or recent debate on whether a putative proof of Fermat's Last Assertion was indeed a proof. This, like Cauchy's reported inability or unwillingness to understand the methods of Galois (an example I owe to Kalyan Basu of the Kanpur Institute of Technology, India) casts doubt on the intuitionist assumption that proof is a decidable notion. (An assumption that after all underlies intuitionistic validity of *ex falso quodlibet.*) Conferring upon proof positive

On the present view, strictly deductive, conclusive argumentation is but the small tip of what appears accountably[27] as a largely inductive[28] or enthymematic iceberg. So there are good reasons why a theory of meaning should grow roots or at least maintain a pied à terre in domains of argumentation not dominated by proof positive.

'Enthymeme' is used here in the sense which Burnyeat (1994) recovered as the most likely intent underlying usage in Aristotle and cotemporaneous or earlier sources. First, it is simply a thought offered in consideration and thereby — in a sense still current for the word — an argument. (Thoughts don't get offered idly, nor without a background of assumptions determining more or less tightly their evidential impact.) In Aristotle's diction, *enthumēma* covers explicit argumentation, albeit of modest complexity and rather less than apodeictic stringency: *apodeixis tis*, 'proof of sorts'.[29] By 'argument' I mean the union of these two classes with the extension of proof positive.[30] [31]

Burnyeat maintains that enthymeme should be seen as a discovery no less than apodeictic syllogism proper, i.e. of what I called proof positive. But what are we to make of it, beyond giving its natural history and defending its claims to rationality in virtue of its participation in what is reasonable?

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the lesser status of a Kantian regulative idea underlying a theory of meaning would deprive it of any advantages it could have claimed over a truth-based one to which a similar treatment could be accorded.

[27]There are still no reliable accounts of what goes on inside people's heads at any pertinent level of neuro-psychological description.

[28]Contrasting with 'deductive' as characterized by monotonicity of the consequence relation.

[29]Current usage meaning '(apodeictic) argument with assumptions or steps left implicit' was a later and, (I think: only) in taking for granted hidden strict apodeixis, unduly loaded re-interpretation.

[30]Hence I would not restrict the notion to that which Parsons (1996) explicates.

[31]The ubiquity of, indeed the moral need for, the relaxed conditions of argument outlined for enthymeme in the *Rhetoric* is brought home in Burnyeat (1996). The author opens with Zeno's argument that one must never listen to the other side. For if the first party proves its case, there is no need to stay around for vain attempts to controvert it. And if it does not, judgment must go against it by default: the pleading was mere prattle, tantamount to non-appearance. Against this background, Burnyeat concludes that in the *Rhetoric* Aristotle defends a notion of proof which ties the first arm of Zeno's dilemmatic attack on a most certain rule of civil conduct: Don't give your verdict until you have heard both sides. (When I saw this paper, a modified version of the 1994 treatment, it was some consolation that it should have been Zeno who had in all but slant anticipated the consideration I had naively thought was news.)

Apodeictic syllogism spawned a long and prolific line of development. It gave rise to a theory of meaning for those elements of language that are among the invariants of argument: the 'logical' function words. Enthymeme, as elicited by Burnyeat, has done nothing as yet comparable. Nevertheless, for those of its instances for which the classic

She is of a sallow complexion, so she is[32] pregnant

is representative, stochastic inference, hence probability, provides an ample home.

Its walls are the decision-theorist's most stable constraints on inductive consistency or coherence (cp. Ramsey 1926, Jeffrey 1965 [1983]). Such constraints properly extend those of classical logic. They do not commit one to imputing people with point-valued personal probabilities any more than the assumption of classical logical competence commits one to precise knowledge of which of the myriad possible worlds (corresponding to maximal consistent sets of sentences of the language) one is in.

But do we not jump to conclusions with our 'so', where we could not have walked by standards of proof positive? Well, we do not simply jump for nothing and from nowhere. Something made us jump, from somewhere, and we may have felt we had no choice but to jump one way or the other. The classical way to handle this predicament is the paradigm of Neyman and Pearson (1933) in the properly decision-theoretic, essentially Bayesian reconstruction and development of Wald (1947). Considerations of situational (dis)utility attaching to 'false positives' and 'false negatives' respectively determine posterior probability thresholds for jumping one way or the other. The epistemic weight of evidence ('argument') then pushes us across, one way or another. In the classical case, if for some reason we found our utilities were wrongly assigned, we still have our current posterior, but also new thresholds. (And if we found our priors were wrong, we might still retain our weights of evidence.)

This way of jumping to conclusions is what the industrialized world's production of middle-sized dry goods runs on and our senses, in the light of experimental results, appear to live by. It strikes me as a sounder approach to the problem than a 'default logic' of one sort or another. The decision whether to jump or not, and if so which way, should be affected by the relative costs of *epoché* and what might turn out to be undue haste, *propéteia*. A probabilistic framework comes closer than anything else on the market to integrating beliefs and preferences in general and defensible fashion.

[32] Or as careful reasoners would put it: must be.

It is the Bayesian core of jumping to conclusion that concerns me here: that on which arguments work by transforming odds or dispositions to conduct, antecedently to any acts of 'acceptance' or 'rejection' we might be forced to take, or be panicked into taking.

Field (1977), standing, as it were, an idea of Popper on its head, has shown how personal *sive* subjective probability provides a semantics for classical logic. This answers a foundational issue: one can define relations of entailment in probabilistic terms and implement classical valuations by suitably defined conditional probability functions. However, when explicating 'grasp' of logical words, more ought to be said; in particular with respect to those cases where propositional relations are properly inductive.

To this end I shall offer results to cast some further light on how argumentative-evidential relations that fall short of entailment can (I hesitate to say: must) constrain or even determine our grasp of propositional logical operations, assuming that natural language morphemes admit of a useful association with them.

5. Argumentative Relations

The axioms of the probability calculus suffice for the probability of a proposition to determine the probability of its negation. But the binary connectives are not, in this sense, probability-functional. This suggests that one might obtain a characterization of negation in general and purely argumentative or evidential terms, while looking for weaker constraints on the grasp or usage of conjunction and disjunction in daily discourse.[33]

Personal probability is interpreted decision-theoretically.[34] That, said, we assume as given an algebra of propositions (i.e. of subsets of a sample space of points such as possible worlds) with elements $A, B, ..., H, ...$ and the set of probability functions on that algebra. Let $P(\cdot)$ be an arbitrary element of this function set. Conditional probabilities are related classically and familiarly to unconditional ones as $P(A|B) = P(AB)/P(B)$ for $P(B) > 0$, else undefined.

[33]Unique argumentative-evidential characterizations for the special case of entailment will be pedantically paraphrastic of logical properties, e.g. that AB is the weakest X entailing, and $A \vee B$ the strongest Y entailed by, both of A and B. This will be uninteresting if grasp over and above Field/Popper-representability is the issue.

[34]The extent to which experimental work on probability assignment (Kahneman et al. 1982) bears on the issue of 'grasp' will therefore have to be addressed. Suffice it to mention that the two best known findings in that line of research — base-rate neglect (the Taxi Cab Problem) and conjunction fallacy (Linda the Feminist Bank-Teller) — can both be rephrased thus: Subjects act as if attending only to evidential relevance of news reports. I do not, hence, think that these findings diminish the interest of characterizing 'grasp' of logical words in non-deductive, argumentative terms.

Qualitative stochastic evidential relevance, say of A to H, which explicates argumentative relations 'for' and 'against' is introduced as a relation between $P(AH)$ and $P(A)P(H)$. 'Neither for nor against' thus amounts to the Kolmogorov definition of independence, and the relation is defined for all values of $P(AH)$, $P(A)$, and $P(H)$. For intuitiveness, however, I give definitions in terms of conditional probabilities where no problems of undefinedness arise acutely.

DEFINITION: A is an argument [i. for] [ii. against] H with respect to a probability measure $P(\cdot)$ iff conditioning on A [i. raises] [ii. lowers] the probability of H i.e. iff
$P(H|A) > P(H) \quad [P(H|A) < P(H)]$

Clearly, when $P(H|A) = P(H)$, we should say that A is neither an argument for or against H. And when $P(A)$ happens to be 0 and $P(H|A)$ is therefore undefined, we should, in agreement with the basic Kolmogorov definition of independence, say the same.

It is fairly obvious that this definition, involving a relation between two probabilities, lends itself to being turned into a definition of a numerical function that represents direction and strength of argumentative force, i.e. of relevance. Two that spring to mind immediately are $P(H|A) - P(H)$ and $P(H|A)/P(H)$. They represent the argumentative or evidential support a proposition A (intuitively considered as evidence and often therefore labelled E) provides for a proposition H (intuitively considered as hypothesis). There are many such functions in use and, on the intended interpretation of numerical values they do, of course, agree with respect to purely ordinal properties, i.e. positive and negative relevance and[35] and irrelevance. They do differ in cardinal properties and certain intuitions about which aspects of the evidential relation they best capture.

The relevance function I shall use in later discussion is one of those which represents the relevance of its arguments (relative to a given probability measure) in such a way that irrelevance maps to 0, and positive and negative relevance map, respectively to positive and negative values. One of the features that distinguish it from the abovementioned functions is that relevance of A to H is not a function of the probabilities of either A or H. Hence, the extreme cases (entailment of H or of $\bar{H}$ by A) receive values independent of the probabilities of the A and H.[36] This class of functions is given by the logarithm (to any fixed base) of the 'Bayes

[35] Given routine conventions which equate absence of positive or negative relevance with irrelevance.

[36] And thus we satisfy the intuition that (non-vacuous) entailment should be representable as a special case of positive relevance.

factor' or likelihood-ratio for non-composite H, i.e. of $P(A|H)/P(A|\bar{H})$. With some proponents' names added it is introduced by the

DEFINITION: The Peirce/Jeffreys/Turing/Good (PJTG) *relevance* of a proposition A to a proposition H is given by the real number $r_H(A) =_{df} log[P(A|H)/P(A|\bar{H})]$.[37]

Thus we note, for future reference as a

DEFINITIONAL FACT: $r_H(A) > / = / < 0$ when $P(H|A) > / = / < P(H)$, and hence $sgn[r_H(A)] > / = / < 0$ when $P(H|A) > / = / < P(H)$.

Return now to argumentative relations as introduced above with another

DEFINITION: A, B are argumentative contraries with respect to H and $P(\cdot)$ iff either one of A, B is an argument for H if the other is an argument against H.

DEFINITION: A, B are universal argumentative contraries iff, for all H and $P(\cdot)$, one of A, B is an argument for H if the other is an argument against H.

FACT: Logical contraries need not be argumentative contraries.
Proof: Construct a countermodel where A, B, H are such that
$A \models \bar{B}$, $A \models H$, $B \models H$. □

Universal argumentative contraries cannot both be arguments for, or both be arguments against, the same proposition. But it may well be that one of them is an argument neither for nor against H. If so, neither is the other one. Now, any algebra of propositions has two elements which have this property for *all* elements H: Ω (tautology) and ϕ (contradiction), the 1 and 0 of Boolean 'multiplication', i.e. of '$\wedge$' here represented by juxtaposition. Now opposites meet in irrelevance to yield an explication of Wittgenstein's Tractarian (1922:4.461)

REMARK: Tautology and contradiction say nothing [and] are senseless.

Explication:
$\forall H, P(\cdot) : P(\Omega H) = P(H) = 1P(H) = P(\Omega)P(H)$
$\forall H, P(\cdot) : P(\phi H) = P(\phi) = 0 = 0P(H) = P(\phi)P(H)$.

The explication distinguishes Ω and ϕ 'dynamically' when interpreted in terms of conditionalization. We cannot update by 0; and updating by 1

[37] Remember: The canonical intuitive interpretation is of A as evidence and H as a hypothesis at issue. Then, by Bayes rule and the logarithmic transformation of multiplicative into additive form, the log of the posterior odds, $P(H|A)/P(\bar{H}|A)$, is the sum of log of the prior odds, $P(H)/P(\bar{H})$, and of $r_H(A)$.

is the identity update. Hence (recalling the relevance function), $\forall H, r$: $r_H(\Omega) = r_H(\phi) = 0$, and thus $\forall H, r : sgn[r_H(\Omega)] = -sgn[r_H(\phi)] = sgn[r_H(\phi)] = -sgn[r_H(\Omega)]$.

6. Argumentative Characteristics of Logical Connectives

The next objective is to see to what extent classical connectives can be characterized in terms of argumentative — and in general non-conclusive — relations which their argument(s) and their value bear towards an arbitrary proposition H that is not, in general, identical with any of the argument(s) and value proposition.

6.1 Negation

Classical negation is definable in terms of the notions 'for' and 'against'. This is not a matter of endocentric relations alone, i.e. that a proposition A will always speak against its complement $\bar{A}$ and for the complement of the complement. (This condition would be satisfied if we generalized 'contradictory' to 'logical contrary'.). The relation is, more often than not, exocentric.

PROPOSITION: For any (distinct) A, B: $B = \bar{A}$ iff, for all $H, P(\cdot)$ any one of A, B is an argument for H only if the other is an argument against H.

Proof: '$\rightarrow$' (Lemma 1), '$\leftarrow$' (Lemma 2).

(Note the import of 'the other', i.e. we need to ensure $A \neq B$ when A is ϕ or A is Ω for the '$\leftarrow$' implication to hold.)

LEMMA 1: $\forall A, B$: If $B = \bar{A}$ then $\forall H, r : sgn[r_H(B)] = -sgn[r_H(A)]$.

Proof: Case " $>$ ": We show $\forall A, H, r : r_H(A) > 0$ iff $r_H(\bar{A}) < 0$.

$r_H(A) > 0$
iff $P(A|H) > P(A|\bar{H})$
iff $P(AH)/P(H) > P(A\bar{H})/P(\bar{H})$
iff $[P(H) - P(\bar{A}H)]/P(H) > [P(\bar{H}) - P(\bar{A}\bar{H})]/P(\bar{H})$
iff $1 - P(\bar{A}H)/P(H) > 1 - P(\bar{A}\bar{H})/P(\bar{H})$
iff $P(\bar{A}H)/P(H) < P(\bar{A}\bar{H})/P(\bar{H})$
iff $P(\bar{A}|H) < P(\bar{A}|\bar{H})$
iff $r_H(\bar{A}) < 0$.

Thus, $\forall A, H, r : sgn[r_H(\bar{A})] = -sgn[r_H(A)]$.

Proofs for cases " $<$ " and " $=$ " are analogous. □

Remark: If $A \in \{\phi, \Omega\}$, then $sgn[r_H(A)] = 0 = -sgn[r_H(A)]$.

LEMMA 2: $\forall A, B, r$: If $\exists H : sgn[r_H(A)] \neq 0$ or $sgn[r_H(B)] \neq 0$ then if $\forall H sgn[r_H(B)] = -sgn[r_H(A)]$ then $B = \bar{A}$.

Proof: (MT, by cases)
1. $B \neq \bar{A}$ (supposition, contrary to consequent)
Case I: $[A \neq \phi \neq B; A \vee B \neq \Omega]$
2. Let $H* = A \vee B$.
3. $P(H * |A) = 1 > P(H*)$; $P(H * |B) = 1 > P(H*)$ [I,2].
4. $r_{H*}(A), r_{H*}(B) > 0$. [I,2]
5. $sgn[r_{H*}(B)] \neq -sgn[r_{H*}(A)]$. [4]
6. $\exists H : sgn[r_H(B)] \neq -sgn[r_H(A)]$. [5] $\surd$.
Case II: $[AB \neq \phi; A \neq \Omega \neq B; A \vee B = \Omega]$
7. Let $H* = AB$.
8. $P(A|H*) = 1 > P(A); P(B|H*) = 1 > P(B)$.
9. $r_A(H*), r_B(H*) > 0$.
10. $r_{H*}(A), r_{H*}(B) > 0$.
11. $sgn[r_{H*}(B)] \neq -sgn[r_{H*}(A)]$.
12. $\exists H : sgn[r_H(B)] \neq -sgn[r_H(A)]$. $\surd$.
Case III: $[A = \phi \neq B; A \vee B \neq \Omega]$
13. Let $H* = A \vee B = B$.
14. $P(A|H*) = 0 = P(A); P(H * |B) = 1$.
15. $r_A(H*) = 0; r_{H*}(B) > 0$.
16. $r_{H*}(A) = 0; r_{H*}(B) > 0$.
17. $sgn[r_{H*}(B)] \neq -sgn[r_{H*}(A)]$.
18. $\exists H : sgn[r_H(B)] \neq -sgn[r_H(A)]$. $\surd$.
Case IV: $[\phi \neq A \neq \Omega = B]$
19. Let $H* = AB = A$.
20. $P(H * |A) = 1 > P(H*); P(H * |B) = P(H*);$.
21. $r_{H*}(A) > 0; r_{H*}(B) = 0$.
22. $\exists H : sgn[r_H(B)] \neq -sgn[r_H(A)]$. $\surd$.
Case V: $[A = B = \phi]$
23. $\forall H* : r_{H*}(A) = 0 = r_{H*}(B)$.
24. 23 fails to satisfy [hypothesis] $\surd$.
Case VI: $[A = B = \Omega]$
25. $\forall H* : r_{H*}(A) = 0 = r_{H*}(B)$.
26. 25 fails to satisfy [hypothesis] $\surd$.
Case VII: $[A \vee B = \Omega; AB = \phi]$
27. $B = \bar{A}$.
28. ruled out by [1] $\surd$.
Hence (by MT)
29. If $\exists H : sgn[r_H(A)] \neq 0$ or $sgn[r_H(B)] \neq 0$ and $\forall H$: $sgn[r_H(B)] = -sgn[r_H(A)]$ then $B = \bar{A}$ [1,6,12,18,22,24,26; MT]
□

The Proposition is jointly entailed by the two lemmata. Hence, in view of our definitions, we have as a

COROLLARY: Distinct A, B are universal argumentative contraries iff $B = \bar{A}$.

and we can conclude with a fairly obvious

REMARK: 'Contraposition' ($A \vdash B$ iff $\bar{B} \vdash \bar{A}$) is a special case of the Proposition.

A useful spinoff from the proposition is the realization that

'He made this move either because he is stupid or because he is not stupid'

cannot reasonably have 'not' delivering the contradictory of the clause in its scope. If you think on, you also have a clue to the meaning of 'at all'. And if these intuitions agree with yours they also provide some tangible support for the idea that a semantics for 'because' satisfies constraints of evidential relevance.

6.2 Conjunction

Models are readily constructible where A, B are arguments for H, but AB is an argument against H, generating a 'paradox of relevance'. (Suppose, e.g. that each of A, B lies mostly in H, but that their intersection in H, i.e. ABH, is empty, while $AB\bar{H}$ is non-empty.)

However, there is a condition linking A, B for any given H that ensures freedom from paradox and, for relevance functions representing sufficient statistics such as the r-function introduced above, full compositionality of relevance.

DEFINITION: A and B are *independent conditionally* on H and $\bar{H}$ with respect to $P(\cdot)$ $[(A \perp B| \pm H)]_p] \Leftrightarrow P(AB|H) = P(A|H)P(B|H) \wedge P(AB|\bar{H}) = P(A|\bar{H})P(B|\bar{H})$.

Intuitively, $(A \perp B| \pm H)$ when knowing whether H or $\bar{H}$ accounts fully for any interactions between A and B.

For conjunction, conditional independence ensures additivity of argumentative weight for the class of relevance functions $r_{(\cdot)}(\cdot)$ defined above, i.e. compositionality of relevance:

FACT: If $[(A \perp B| \pm H)]_p]$, then $r_H(AB) = r_H(A) + r_H(B)$.[38]

[38] The proof is as obvious as the fact well known. Without any independence assumptions we have as a FACT: $r_H(AB) = r_H(A) + r_{H|A}(B)$.
The epistemic dynamics here parallels analyses of context-change familiar from the literature on dynamic approaches to discourse.

> COROLLARY: Given conditional independence w.r.t. H, if each of A, B is an argument for H, then AB is an argument for H and indeed, if neither of A, B entails H (when the function goes to infinity), an argument strictly stronger than either of A, B.

This is far from giving a full characterization of conjunction (recall: we are not interested in characterizing just the special case of entailment). However, it suggests that our 'grasp' of conjunction will be informed by prototypical evidential constellations that make for conditional independence.

Intuition 1 for such a constellation (Reichenbach 1954): H is a common cause of A and B. I.e., we advance propositions A and B as arguments for the truth of a proposition H that characterizes such a cause.[39]

Intuition 2 (a special case, one might say): An event H of which A and B are reports by distinct, potentially unreliable, but independent (non-colluding, individually observing) witnesses.

Indirect evidence for a default assumption of independence conditional on issues at hand, i.e. of local relevance-functionality of '$\wedge$', is the prevalence of dichotomic issues, i.e. of issue partitions $\{H, \bar{H}\}$. For partitions $(H_i : i = 1, \ldots, n > 2)$, at most one evidential proposition deemed atomic can be relevant to any given H_i if conditional independence of evidential atomic propositions is to be satisfied w.r.t. each element of the issue partition and its complement (Johnson 1986).

Undesirable as such a constraint may be for the cognitive engineer it suggests an empirical hypothesis. Suppose the Maker of human minds values additivity and, a fortiori, freedom from paradoxa of relevance. This will suggest a strong tendency for restriction of issue-partitions ('competing theses') to the case $n = 2$. Anecdotal evidence speaks for such a bias being evolutionary fact. Of course, more common-sense reasons for it can be thought of. But on what grounds do we *trust* common sense?

The argumentative view also suggests that cognitive categories — and that means: our language — might be structured in such a way as to minimize the incidence of 'paradox'. Spoof diagnostic examples of paradox are easy to construct, as are examples involving causal intervention. Yet it seems difficult — and for physicians I have queried over drinks,[40] impossible — to find examples of bona fide medical diagnostic categories yielding a syndrome H such that each of clinical observations A and B individually would speak for H, but AB against.

[39] Propositions are here usefully thought of as properties of events.

[40] And another, with a statistical background, queried via e-mail by a seasoned probabilist

6.3 Disjunction

A 'paradox' perhaps more counterintuitive yet can be obtained: A, B each speak for H, but $A \vee B$ against (cf. e.g. Carnap 1950 or just imagine a situation whose most salient feature is $AH = BH$ while $A\bar{H}$ and $B\bar{H}$ are disjoint). Clearly, in such a case a fairly robust intuition about the argumentative force or weight of a disjunction will also be violated: namely that the relevance of $A \vee B$ be a convex combination of (i.e. lie between, equivalently be a 'mixture' of) the relevances of A and B individually.

Conditional independence guarantees freedom from paradox. However, it will not ensure convexity. Here a simply stated condition guarantees both for relevance functions defined by quotients of conditional probabilities. Instances are Johnson/Keynes' $P(XY)/P(X)P(Y)$ 'coefficient of influence' transforming prior into posterior probability by multiplication; or the Bayes factor, doing the same for odds:

PROPOSITION: For any A, B, H and $P(\cdot)$: If $P(AB) = 0$, in particular if A and B are presuppposed to be disjoint, then the (Bayes factor or J-K coefficient) relevance of $A \vee B$ is a convex combination of the relevances of A and of B. (Hence, since logarithm is continuous monotone, also on taking logarithms, e.g. for PJTG function.)

Proof: I. (Log-)Bayes Factor:

1. $P(AB) = 0$.

2.$P(A \vee B|H)/P(A \vee B|\bar{H}) = [P(A|H) + P(B|H)]/[P(A|\bar{H}) + P(B|\bar{H})]$ [1]

$= [P(A|\bar{H})/[P(A|\bar{H}) + P(B|\bar{H})]] \cdot [P(A|H)/P(A|\bar{H})] + [P(B|\bar{H})/[P(A|\bar{H}) + P(B|\bar{H})]] \cdot [P(B|H)/P(B|\bar{H})]$

$= \alpha[P(A|H)/P(A|\bar{H})] + (1-\alpha)[P(B|H)/P(B|\bar{H})]$ where $\alpha \in [0, 1]$.

3. Hence $r_H(A \vee B) = \beta r_H(A \vee B) + (1 - \beta) r_H(A \vee B)$ for some $\beta \in [0, 1]$. □

II. J-K Coefficient (see below).

For many contexts of use this constraint[41] affords a simple explanation of the much discussed intuition that 'or' is read as exclusive, albeit defeasibly so.

This mixture property underlies a result showing in which almost literal way argumentative relevance explicates the classical rhetorician's *utilitas causae*:

[41] A general condition Boole (1854) placed on definedness of his logical '+' operation, and not to be confused with the assumption that an assertion of 'A or B' is a statement of the exclusive (XOR) disjunction. See Merin (1997) for comparative relevance of OR and XOR statements.

PROPOSITION: Let $P(\cdot), H$ be arbitrarily fixed. Then, for any A, $f[H,P](A) =_{df} P(AH)/P(A)P(H)$ is a cardinal utility.
Proof: Let $P(AB) = 0$ and $P(H), P(A \vee B) > 0$.
Then $P(H(A \vee B))/P(H)p(A \vee B) = [P(AH) + P(BH)]/[P(A)P(H)+P(B)P(H)] = [P(AH)+P(BH)]/[[P(A)+P(B)]P(H)] = [1/[P(A)+P(B)]] \cdot [P(A)[P(AH)/P(A)P(H)] + P(B)[P(BH)/P(B)P(H)]]$.
Thus, where disjuncts are disjoint, we have the relevance of the disjunction as a probability-weighted mixture of its disjuncts. I.e. it satisfies, where $u(\cdot) = r_H(\cdot)$ for given H, the Axiom (Jeffrey 1965: 5-2):
If $P(A \vee B) \neq 0$ and $P(AB) = 0$
then $u(A \vee B) = [P(A)u(A) + P(B)u(B)]/[P(A) + P(B)]$.
This makes the relevance function a cardinal utility function (intuition: the utility of one of its arguments in a context represented by its other argument) defined at least up to a quotient of positive affine transformations (Jeffrey 1965, Chs. 5 and 6). □

PROPOSITION: Given conditional independence $(A \perp B| \pm H)$ non-entailment of H by either of A, B, $r_H(A)$ and $r_H(B)$ are cardinal utilities in the sense of Jeffrey (1965).
Proof Sketch: We can show vacuous satisfaction, by antecedent failure, of Jeffrey's axiom (5-2). For if $(A \perp B| \pm H)$ and $P(AB) = 0$ then A must entail one of $\{H, \bar{H}\}$ and B the other. Hence, given the antecedent of the proposition, $P(AB) \neq 0$ and the condition under which the utility of the disjunction must be a probability weighted mixture of disjuncts is never satisfied. □

These results on additivity and convexity under suitable and intuitive conditions suggest that one might look for a semantics for sentences of English in which coordination (and by implication quantification) proceeds in a way that takes these properties for granted.

7. Indexical Semantics in Linear Algebras

The standard explication of a semantics for well-behaved languages and formal fragments of natural ones is a homomorphism from a syntactic to a semantic algebra. Given the advantages of compositionality, computing languages and formal fragments of natural-like languages have striven to make use of it. For the latter, truth-conditions or truth-like conditions have provided the backbone of direct (Keenan and Faltz 1985; Montague 1970) or ultimate, post-dynamic (Kamp and Reyle 1993) interpretation.

What, now, if one followed the algebraic thesis expounded in Sections 1 to 3 of Montague (1973) — i.e. the truly universal bit couched entirely in terms of universal algebra — and then replaced *truth* — the specific turn taken in Section 4 — by an economist's pragmatic notion of *'use'* for the purpose of inducing semantic recursion?

Recall that boolean structure can be lifted pointwise from the smallest non-trivial *boolean algebra*, the space $(\mathbf{2}; \vee, \wedge, ')$ of truth-values, to the function space $\mathbf{2}^I$, where I is an arbitrary non-empty set. This makes $\mathbf{2}^I$ a boolean algebra too. If I is a set of possible worlds, $\mathbf{2}^I$ is the associated space of propositions in classical truth-conditional semantics — and so on for subsentential recursion (functions from individuals into propositions, etc.).[42] Empirical grounds may then dictate choice of sub-algebras of homomorphisms for denotation spaces (Keenan and Faltz 1985).

To yield semantic structure in its own right—rather than as a Gricean afterthought predicated on some antecedent semantics—'use' will also have to be explicated in terms of an algebra of ultimate values. If so, the natural choices of explicata are cardinal *utility* (in the sense of Ramsey, von Neumann/Morgenstern, or Jeffrey) and—as a quasi-utility for ordinary indicative-mood discourse—evidential *relevance* measured by the log-likelihood-ratio explicating *valeur argumentative* (Ducrot 1973).[43]

These have for range the space $(\mathbf{R}; +, \cdot)$ of real numbers, which is a *linear algebra* (over itself). Hence, semantic algebras, say the space $\mathbf{R}^C$ of functions from contexts-of-use C to $(\mathbf{R}; +, \cdot)$ will be linear algebras. And so on, analogously for subsentential coordination.

Whether or not one ultimately relies on the pointwise lifting construction: a semantics in linear algebras over $\mathbf{R}$ seems worth exploring for a possible mental (psychological?) level of semantic representation, antecedent or parallel to construction or induction of logical forms.[44] The general idea is perhaps a way to give a non-nihilist answer to a question arising with the late E.J. Lemmon's (1965) suggestion, that sentences of natural languages do not have logical forms, while arguments conducted by means of them do.

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[42] From a structural point of view, the important thing about Truth and Falsity is that they are two and carriers of a certain algebra.

[43] See him for revolutionary, insightful informal treatments of semantic phenomena no-one considers to lie within the ambit of truth-conditions. Check Merin 1995, 1997 for expositions of some of his insights and, on the formal side, for evidence that natural languages obey inductive constraints postulated by Carnap and engaged by the theorem of Savage, Kemeny, Gaifman, and Humburg.

[44] In the logician's sense of logical form.

Collateral motivation might be the eminent role of largely linear systems in psychological models of perception, memory, learning and rational conduct. But, of course, the most important type of evidence for the would-be student of language will be descriptive success.[45] We must therefore specify some fragment of such a semantics which will warrant play for appreciable stakes. I shall here concentrate on coordination—a topic many philosophers find unproblematic but which harbours difficulties not generally appreciated, even if we disregard problems arising in other than straightforward assertoric contexts.[46]

The elements of a boolean algebra are boolean polynomials of generating elements. Analogously, elements of a linear algebra (vector space) are *linear combinations* $\sum \alpha_i \mathsf{x}_i$ ($\alpha_i \in \mathbf{R}$) of generating vectors x_i. Leaving aside for now the ontological status of such vectors,[47] this leads to the following

HYPOTHESIS: Binary coordination denotes *indexical linear combination*. That is to say: Let

- $\mathsf{x}, \mathsf{y} \in X$ be vector variables, where X ranges over denotation spaces of conjoinable categories;
- $\langle c^\circ, k, w \rangle$ be a triple of indices standing for ⟨utterance-context, k-th occurrence-in-cotext, world⟩, where the occurrence-index intuitively individuates tokens of a connective (say 'or') in a piece of discourse under consideration, here a sentence; utterance-context and world(-state) are intuitively as in familiar versions of intensional semantics (cp. the context-tuples in Lewis 1972);
- $\tilde{c} := \langle c^\circ, k, w \rangle$ be an abbreviation;
- α, β be functions from the space of such context-triples into the real numbers, such that
- $\alpha_{\tilde{c}} = \alpha(\tilde{c}) = \alpha(c^\circ, k, w) \in \mathbf{R}$, and $\beta_{\tilde{c}} = \beta(\tilde{c}) = \beta(c^\circ, k, w) \in \mathbf{R}$.

Then an *indexed linear combinator* is a function

$$\lambda \tilde{c} \mathsf{xy}[\alpha_{\tilde{c}} \mathsf{x} + \beta_{\tilde{c}} \mathsf{y}]$$

such that for

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[45] The sure way to prove a philosopher of language wrong is to show that he gets either his mathematics (however rudimentary) or his facts wrong. A philosopher who is reluctant to be drawn on either count is, to speak with Austin, not sticking his neck out.

[46] Recall Section 2.

[47] For denotata of sentences the answer to the analogous ontological question in truth-conditional semantics is that denotata are sets of possible worlds; in proof-conditional semantics, that they are sets of proofs. I shall, very briefly, return to the question later on.

- distributing *and*: $\alpha_{\tilde{c}} = \beta_{\tilde{c}} = 1$; i.e. simple vector addition.[48]
- non-distributing 'collective' *and*: $\beta_{\tilde{c}} = 1 - \alpha_{\tilde{c}} \in]0, 1[$;[49]
- *or*: $\beta_{\tilde{c}} = 1 - \alpha_{\tilde{c}} \in \{0, 1\}$, such that α is a random variable ostensibly under the control of someone other than the Speaker: either (i) the addressee [for imperatives] or (ii) Nature (or a third party) [for simple indicatives].[50] Note:
  1. $\alpha_{\tilde{c}} = 0$ means not 'x is false'; but rather non-commitment to eventual supply of evidential or other backing for x.
  2. for $k \neq k'$, we have no constraint $\alpha(c^\circ, k, w) = \alpha(c^\circ, k', w)$; indeed we must expect obviation, just as in familiar pronominal phenomena.

Does one need indexical connectives? For a linear semantics, necessarily so (see definitions, and exx. below). And dissatisfaction with the descriptive powers of operator 'scope' in logical form[51] might suggest independent grounds for wanting it. (I shall not go into the issue here.) An example that can still be handled with scope by using type-raising is Rooth and Partee's (1983:374)) famous *Kim is looking for a phonologist or a phonetician*.[52]

Within the present framework (though certainly also much more intuitively) the natural approach yielding the example's three prominent distinct readings is one where allocation of choice required for interpreting *or* resides, respectively, with Speaker (who boorishly refuses to come clean), Kim, and Nature. As in the case of Free Choice Permission this description captures directly intuitions of choice allocation (cp. the Latin imperative *vel!*) which a Fregean scope analysis will have to project onto its proof and model theory *ad hoc*.

Our formalism accomodates the prohairetic element quite unmessily. Linear algebras, where scalars α_i are technically operators on the additive group of the vector space, offer a mathematically clean way, *sans bricolage*, of making connectives *indexical*. Unlike lattice algebras.

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[48] Note that, unlike lattice-join (e.g. boolean conjunction), vector addition, of which a special case is addition of real numbers, is not an idempotent operation.

[49] This will be illustrated in the subsection of subsentential recursion: basically it pertains to lexically non-contradictory attributes such as 'black and white' or collectives of individuals denoted by, say 'Kim and Sandy' who ('between them', as English so geometrically puts it) own $5, or manage to carry the piano upstairs.

[50] This explicates the familiar convention of speaker's inability/unwillingness/non-commitment to identify the disjunct (to be) realized.

[51] Fauconnier (1984:86ff.) argues that for Ioup's variant on Geach's 'Hob-Nob' sentences, viz. (I simplify) *Everyone believes that a witch has blighted Fido*, there are distinct readings conflated, and one not captured at all, by scope analyses.

[52] To cut away inessentials I have substituted 'Kim' for 'The department'.

The example just mentioned was introduced for heuristic purposes only. To say anything really serious about attitude contexts would mean having a semantics for attittudes, which, in the present framework, I have not. But already at the level of simple sentential coordination the analysis can show not only correspondence with intuitions but tangible advantages over the received analysis.

7.1.1 Sample Problem: Where Meet and Join Don't Meet or Join

Boolean and weaker lattice semantics face little-known, but highly robust[53] problems such as curious asymmetries of acceptability and interpretability for the absorption and distributive laws. Thus consider:

(1) *Al is heavy or Al is heavy.
(2) *Al is heavy and Al is heavy.
(3) Al is heavy, or Al is heavy and Cy is tall.
(4) ??Al is heavy, and Al is heavy or Cy is tall.
(5) Al is heavy.
(6) Al is heavy, or Bo is slim and Cy is tall.
(7) ??Al is heavy or Bo is slim, and Al is heavy or Cy is tall.
(8) Al is heavy, and Bo is slim or Cy is tall.
(9) Al is heavy and Bo is slim, or Al is heavy and Cy is tall.

Boolean interpretations $||or|| = \vee$, $||and|| = \wedge$ demand the following equalities

- $||(1)|| = ||(5)|| = ||(2)||$ (by the laws for semilattice algebras);
- $||(3)|| = ||(5)|| = ||(4)||$ (by the laws of lattice algebras); and
- $||(6)|| = ||(7)||$, $||(8)|| = ||(9)||$ (by the laws for distributive lattices).

The unacceptability (except in heavily sarcastic usage) of (1) and (2) is routinely noted and put down, plausibly, to 'redundancy' of one kind or another.The very low acceptability and interpretability ratings for ??(4) and ??(7) are completely overlooked.[54] On the other hand, as anyone may check simply by muttering (4) and (7) aloud and comparing with mutterings of their respective duals (3) and (9), they are highly robust.

Once pointed out they are often attributed to Gricean pragmatics. But no such account is actually forthcoming. (And I have tried many people in the field over the last few years.) 'Redundancy' ("don't use A if $A \leftrightarrow B$ and B is shorter") does look like explaining *(1), *(2). But

[53] They won't go away if you stare at them long enough, nor will they if others do.

[54] The closest the literature comes to acknowledging them is Keenan and Faltz (1985), which gives examples of the unproblematic cases but smartly leaves the dodgy ones as routine exercises in checking equivalences, logical training being presupposed throughout.

if we take it seriously, we should note that it also predicts ??(3), ??(9). 'Supplementation' of leftmost A to $A \wedge \bar{C}$ looks like explaining ??(4) but won't do for ??(7). Nor will the implicature algorithms of Gazdar (1979) or even quite outrageous liberalizations of them.[55]

7.1.2 Linear Account

In what follows, $[\![\cdot]\!]$ designates the linear semantic interpretation function. This is the simple part of the notation. But the details of the scalar coefficent scheme require some exposition. The first part of this concerns their indexical aspect.

Recall that $\kappa_{\tilde{c}} := \kappa(c^{\circ}, k, w)$ represents the way which scalar coefficients (i.e. real numbers) for indexical linear combination are represented, where κ instantiates to $\alpha, \beta, \ldots$. Assume now that the context of utterance, c°, referred to for computing all example paraphrases stays constant. Thus, two components of $\tilde{c}$ may vary: the occurence-parameter, k, and the world or, if you will, evidential supply/support parameter, w. Let us now adopt the further convention that k stays constant for any given instantiation $\alpha, \beta, \gamma, \ldots$ of the function variable κ.

Since there is no empirical reason to assume that our examples involve non-distributing 'and'[56] the two scalar coefficients associated with 'and' are always unity and can be omitted. So 'and' is not, in the present cases, indexical. By contrast, 'or' is indexical, though, of course subject to the constraint that the value of the second coefficient is unity minus the value of the first and either 0 or 1. Thus, each occurrence of 'or' can be associated with a single coefficient $\alpha, \beta, \gamma, \ldots$ which is, in turn, a function of w and which can take values in $\{0, 1\}$. For visual ease I index each occurrence or 'or' within examples with a distinct Greek letter $\alpha, \ldots$, e.g. or_{α}. In metalinguistic specification of values, κ_w stand for the value of the coefficient κ for a particular instantiation w of the world-coordinate. I use $=_w$ to designate 'extensional' equivalence at w; i.e. equivalence for some w characterized by a particular value for κ_w. When checking for conditions of possible equivalence cutting across expansions in individually numbered examples (say, for general equivalences such as those predicted by lattice-algebraic laws between (6) and (7)) or when noticing local, extensional equivalences for certain values which are not so predicted, I employ the familiar notation $[\beta/\alpha]$ for alphabetic substitution of symbol α for symbol β, to make comparison possible.

[55] See my very pedantic 60pp Ms. "Where Meet and Join Don't Meet or Join", Stuttgart 1994.

[56] For one, they involve no essentially sub-sentential, i.e. phrasal coordination.

The next part concerns the interpretive consequences of the fact that distributing 'and' designates vector addition, which is non-idempotent. Recall the idea that our space of ultimate values is the codomain of a utility- or relevance-valuation. Now, a truth-valuation in classical propositional logic is not just any function from the set of sentences into $\{0, 1\}$ or its alias $\{T, F\}$, but rather one that preserves truth-functional recursive structure. In other words, it is a homomorphism of boolean algebras. Thus, in the light of analogy and of what it means to be a semantics, utility or relevance valuations should be homomoprhisms of linear algebras. Since their codomain is the underlying field (the reals), they will be—in standard terminology—linear functionals. (A pretty good model of a linear functional is a restaurant bill.) Let v be such a valuation. Then it should satisfy, for sentences A, B, mapped to denotata in a linear space,

$$v([\![A]\!] + [\![B]\!]) \;=\; v([\![A]\!]) + v([\![B]\!]),$$

i.e. additivity. For v to be interpretable as a utility or relevance function in the standard unitary sense of Jeffrey (1965) it should have as its domain a boolean algebra of propositions. The way to represent there that both A and B are realized is, of course, to represent their conjunction as being realized, or the intersection of their set-algebraic denotata. With respect to that algebra and a real-valued function v' defined on it, additivity comes to

$$v'(|A| \cap |B|) \;=\; v'(|A|) + v'(|B|).$$

and carries a requirement of independence. A typical counterexample for utilties is getting a left shoe and getting a right shoe. For relevance, where v' is computed as with respect to some contextual H, conditional independence is the requirement. Consider now relevance. There are two conditions where this requirement cannot possibly be satisfied (and hence not even assumed or pretended to be satisfied). This is when $B = A$ or $B = \neg A$. The second case, of contradiction, is familiar. And ordinary usage, as in *Kim is fat and she isn't fat* gives one a strong secondary intuition thart the way to make sense of this is to have the valuation, as it were, change in mid-sentence. So we do not, in the straightforward sense, have compositionality. In the case where $B = A$ something analogous is much more difficult to intuit, if at all. I shall not try to speculate why. What matters is that the case $B = A$ ensures, except in degenerate cases (tautology) that A and B must fail the requisite conditional independence requirement. When it comes to simply adding relevance as defined by the log-likelihood ratio, then it is clear that additivity for $B = A$ is possible only if the relevance is either zero or infinite. The latter, given regularity of the probability measure involved, holds only when A entails H, i.e. is a conclusive argument for H.[57]

For 'or' a condition that the speaker be ignorant of which disjunct is (to be) realized is familiar from Grice and from Schröder (1890:I,134). Griecan derivations of it appeal not just to informativeness, but also to the Maxim of Manner, enjoining brevity of expression. I shall not go into the issue here (see my 1997 paper for that). But note that preferential, poorly defeasible, allocation of choice to someone other than the speaker would also make for ridiculousness if the choice turned out to be as in the sarcastic 'You can pay up or you can pay up'.

Consider now our example sentences. By the laws of linear algebra and our definitions we get the following ('L' for 'linear') corresponding analyses:

(1) L $[\![A \, or_\alpha \, A]\!] = \alpha[\![A]\!] + (1-\alpha)[\![A]\!] = [\![A]\!]$, but violates the choice condition of Def.or, (alternatively, the Schröder/Grice condition).

(2) L $[\![A \, and \, A]\!] = [\![A]\!] + [\![A]\!] = 2[\![A]\!]$ [!!]; this violates the additivity constraint (for non-iterables); while $(2L) = (5L) \neq \mathbf{0}$ (the putative 'idempotent' equivalence under) is unsatisfiable.

(3) L $[\![A, \, or_\alpha \, A \, and \, C]\!] = \alpha[\![A]\!] + (1-\alpha)[[\![A]\!] + [\![C]\!]] =_w [\![A]\!]$ if $\alpha_w = 1$; $[\![A]\!] + [\![C]\!]$ if $\alpha_w = 0$. No acceptability problem, thus. Still, $(3L) = (5L)$, (the putative 'absorption' equivalence) is invalid, though satisfiable.

(4) L $[\![A, \, and \, A \, or_\beta \, C]\!] = [\![A]\!] + \beta[\![A]\!] + (1-\beta)[\![C]\!] =_w [\![A]\!] + [\![C]\!]$ if $\beta_w = 0$, and $=_w 2[\![A]\!]$ if $\beta_w = 1$ [!!] i.e. violates the constraint of Def.*and* for a possible parametrization. Unsatisfiable: $(4L) = (5L) =_{df} [\![A]\!] \neq \mathbf{0}$. But $(4L) =_w (3L)[\beta/\alpha]$ if $\beta_w = 0$.

(5) L $[\![A]\!]$.

(6) L $[\![A, \, or_\alpha \, B \, and \, C]\!] = \alpha[\![A]\!] + (1-\alpha)[[\![B]\!] + [\![C]\!]] =_w [\![A]\!]$ if $\alpha_w = 1$; $[\![B]\!] + [\![C]\!]$ if $\alpha_w = 0$.

(7) L $[\![A \, or_\beta \, B, \, and \, A \, or_\gamma \, C]\!] = \beta[\![A]\!] + (1-\beta)[\![B]\!] + \gamma[\![A]\!] + (1-\gamma)[\![C]\!] = \beta[[\![A]\!] - [\![B]\!]] + \gamma[[\![A]\!] - [\![C]\!]] + [\![B]\!] + [\![C]\!] =_w 2[\![A]\!]$ if $(\beta_w, \gamma_w) = (1,1)$ [!!]; $[\![A]\!] + [\![C]\!]$ if (1,0); $[\![A]\!] + [\![B]\!]$ if (0,1); $[\![B]\!] + [\![C]\!]$ if (0,0). $(7L) = (6L)$ is invalid; we only have $(7L) =_w (6L)[\beta/\alpha]$ for $\beta_w = \gamma_w = 0$.

(8) L $[\![A, \, and \, B \, or_\alpha \, C]\!] = [\![A]\!] + \alpha[\![B]\!] + (1-\alpha)[\![C]\!] = \alpha[[\![A]\!] + [\![B]\!]] + (1-\alpha)[[\![A]\!] + [\![C]\!]] =_w [\![A]\!] + [\![B]\!]$ if $\alpha_w = 1$; $[\![A]\!] + [\![C]\!]$ if $\alpha_w = 0$.

[57] Without regularity: only if it is conclusive to all feasible intents and purposes.

(9) L $[\![A\, and\, B,\, or_\alpha\, A\, and\, C]\!] = \alpha[[\![A]\!]+[\![B]\!]]+(1-\alpha)[[\![A]\!]+[\![C]\!]] = (8L)$.
N.B.: $\not\vdash (7L) = (9L)$, but $(7L) =_w (9L)[\beta/\alpha]$ if $\beta_w = \gamma_w = 0$.

Empirical Hypothesis: The unacceptability judgments ??(4) and ??(7) are due to satisfiability of $(4L) =_w 2[\![A]\!] =_w (7L)$. Hence they obtain for reasons similar to those of *(2): violation of the homomorphism property required for use-valuations. For that violation to bite, the satisfiable equivalence will, I suppose, have to be something which cannot be simply ruled out by some such rule as 'admit only those w that create infelicity'. If the 'meaning' of the sentence is given—in analogy to the way a proposition is thought of as a *set* of worlds—by the set of possible instantiations, then such a tactic would not be admissible, least of all at the poorly manipulable level of repreentation we are addressing.

Note, then, that here boolean idempotence represents a constraint operative subsequent to linear recursion. This is the constraint against counting the same piece of evidence twice (in indicative contexts) or one and the same event twice in imperative contexts where merit or blame is apportioned accordingly.[58] Logic, in particular the idempotent law (which Boole took to be the most distinctive of the normative laws of thought), comes in as a constraint against abuses one might otherwise be able to get away with. Logic has a defensive function, thus — as indeed its historical origins as a conscious discipline suggest. *He$_i$ talks and he$_i$ talks* is an example where there are no problems, i.e. an iterative construction; while *He$_i$ is tall and he$_i$ tall* is unacceptable. The distinction in verbal aspect of the verb phrases encode such differences of interpretability. But what explains, formally, the inadmissibility of the second?

Grican explanation, or any other explanation that induces semantic recursion in boolean or other lattice-algebraic fashion, 'redundancy' of even the simple case *(1) must be a matter of *syntax* somehow out of tune with semantics. For clearly, if we took the semantics seriously, idempotence would simply deliver the same interpretation for (1) as for (5). If we then ask why other morpho-syntactically wasteful ways of expressing the same thing do not engender unacceptability intutions, we should find Gricean redundancy (a violation of 'Be brief!') somewhat unconvincing. Perhaps auxiliary hypotheses invoking implicatures might help us out; but I have not seen any general ones advanced.

Finally, without linear (or similar non-boolean) recursion the idempotence constraint could not bite to explain all three cases. In other words, the semi-theoretical intuition or feeling that these cases are defective in

[58] Analogies: Charging the same plate of soup twice on the restaurant bill. Thinking that being told twice by the same person doubles evidential weight.

a similar way cannot be derived by Boolean, Heyting or lattice-algebraic means at all. (Exercise!)

7.2 Subsentential Recursion

This will be very brief. A quasi-Fregean, categorial way is to interpret functions as linear transformations (vector space homomorphisms) on vector arguments. Such function spaces are closed under vector operations. Hence, no immediate empirical point will hinge on assignment of function / argument roles. (But mediately categoriality will introduce constraints on dimensionality for which I have no independent motivation.) Let sentences denote vectors in a space S, VPs denote vectors $\mathsf{w}, \mathsf{t} \in V$, another linear space; let 'subject' NPs denote linear maps $\mathsf{A}, \mathsf{B} \in Hom_R(V, S)$;[59] let $+ : X \times X \rightarrow X$ be polymorphic, i.e. with X ranging over S, V, $Hom_R(V, S)$ and any other spaces of denotata for conjoinable syntactic categories. Thus we have, for example

(10) $[\![\textit{Al and Bo walked}]\!] = [([\![Al]\!])[\![and]\!]([\![Bo]\!])]([\![walked]\!]) = [\mathsf{A}+\mathsf{B}]\mathsf{w} = \mathsf{Aw} + \mathsf{Bw} = [([\![Al\, walked]\!])[\![and]\!]([\![Bo\, walked]\!])] = [\![\textit{Al walked and Bo walked}]\!]$.

(11) $[\![\textit{Al walked or talked}]\!] = [\![\textit{Al walked } or_\alpha \textit{talked}]\!] = [\![Al]\!]([\![walked\ or_\alpha talked]\!]) = \mathsf{A}(\alpha_{\bar{c}}\mathsf{w}+(1-\alpha_{\bar{c}})\mathsf{t}) = \alpha_{\bar{c}}\mathsf{Aw}+(1-\alpha_{\bar{c}})\mathsf{At} = [\![\textit{Al walked } or_\alpha \textit{ Al talked}]\!]$
$= [\![\textit{Al walked or Al talked}]\!]\, (\alpha_{\bar{c}} \in \{0,1\})$.

(12) $[\![\textit{Fido is black } and_\alpha \textit{ white}]\!] = \mathsf{F}[\alpha_{\bar{c}}\mathsf{b}+(1-\alpha_{\bar{c}})\mathsf{w}] \neq \mathsf{Fb} + \mathsf{Fw} = [\![\textit{Fido is black and Fido is white}]\!] \quad (\alpha_{\bar{c}} \in]0,1[)$.

Linear indexical semantics can be shown to predict correctly for 'double-jointed' coordination *Al* {*and/or*} *Bo walked* {*and/or*} *talked.* Unlike in Boolean accounts (Keenan and Faltz 1983) or extended Montague-Semantics no appeals to function/argument typeshifting and to (here) unmotivated 'collective' readings of *and* are made.

Extension to standard quantifiers is straightforward. The representation of negation—remember, we are talking about natural language negation—is more difficult. Briefly and informally (cf. Merin 1994, 1997 for more), the hypothesis is that negation, as instantiated in denials of imperative or indicative claims, operates on the positive cone with respect to the claimant's ostensible complete preference ranking of locally alternative claims or negotiation outcomes, generated by the denotatum. (Recall that a claim for five dollars never excludes more than, i.e. amounts to a claim for at least, five dollars.) Complementation of that cone yields a

[59] This space of vector space homomorphisms from V to S is itself a linear space. Recall Oehrle's quote.

set glossable 'less than X', where X generated the claim 'at least X'. The same effect can be reconstructed by interpreting *not* as the product of two involutions: one reversing contextual preference orderings, the other toggling equality and inequality.

This brings up the question of what the vectors—in the first place those representing sentence denotata—really are. At present I should like to keep an open mind on that issue. After all, it has long been the case that formalisms were proposed whose ontology came to be interpreted only much later (think of model theory, sets of posssible worlds, etc.). If pressed for a quick answer, I should say they are formal linear combinations of atomic propositions, from which atomic properties and the like are induced. A better answer, to be worked out, would perhaps treat them sui generis as mental representations much more like the ones that look plausible for sensori-motor phenomena, but having epistemic interpretations in virtue of the well-known relation between geometry and epistemic probability.[60] The dynamic, context-changing nature of communication might then be taken care of if we interpret our denotation vector spaces as transformation spaces of a linear space of such representations.

How, then, do we induce truth-conditions? We should do so via the dynamic interpretation, by construing valuations as default parametrized stochastic relevance functions under idealizing default conditions: e.g., conditional independence of conjuncts with respect to the cells of a dichotomic issue. An example of this correlation was given above with respect to 'and'. The explanation for the unacceptability of sentences that should create no problems for a boolean semantics is a case in point: for in such cases conditional independence is never satisfiable for any but the special case of one of the issue propositions being entailed by conjuncts.

Similarly, we have seen that the conditional independence assumption (Merin 1997) guarantees freedom from paradoxes of relevance for disjunction. And, as we have also seen, a *presupposition* (ibid.) of exclusivity of disjuncts guarantees that a disjunction's relevance is a convex combination of disjuncts' relevances.

For 'or' one should thus note that, although the value of the criterial random variable κ is in $\{0, 1\}$, its *expected value*, pertinent for participant

[60]Note, perhaps to excuse such ontological laxness, that at present there are very serious questions even among philosophers committed to formal argumentation about the nature of the objects of belief. Answers such as 'sets of possible worlds' are not now considered to be good answers to those questions.

assessment of its evidential impact, will be given by a subjective probability distribution over those values in the case outlined in Section 6.[61] Thus, we do have convex combination entering what would appear to be the generation of semantic intuitions.[62]

8. Conclusions

This investigation has pursued the consequences of the ubiquity of non-conclusive argumentaton in two directions. First, the question was to what extent the grasp (satsifying the rules of logic) of logical connectives could be reconstructed by taking 'for' and 'against' as the basis of our construals, rather than True or False or Proved and Disproved.

It turned out—unsurprisingly with hindsight—that among the logical operations, classical negation can be defined solely in terms of probabilistic explications of the argumentative notions 'for' and 'against'. [63] This presents an alternative to familiar definitions of negation in terms of notions 'true' and 'false'.

When it comes to logical grasp of binary connectives, argumentation short of proof positive, but explicated within the probability calculus (and assuming $P(A|B) = 1$ to be as good as proof positive of A granted B) cannot, as far as I can see, be a real substitute for proof or truth conditions. Unlike negation, these operations are not probability-functional. Unique characterizations for Boolean conjunction and disjunction can, of course, be given without further assumptions in the above argumentative terms if the special case of conclusive argumentation is considered and paraphrased (cf. Field 1977). But this is quite uninteresting for the present question.

But note that non-conclusive argumentation characterizes logical operations best exactly where intuitive grasp in terms of proof positive is weakest. Thus, in the familiar systems of classical and intuitionistic deduction, negation is the sole connective that is defined either by rules or axioms having two occurrences of the operator, or else by appeal to an elusive object: the absurd or contradictory proposition (semantically:

[61] The 'inclusive' reading would then be induced by bargainer's preference monotonicity, i.e. the 'at least' qualification leaving room for windfall gains.

[62] And, returning to the examples ??(4) and ??(7), we might say that conventional distributions do not assign the constellation that leads to idempotence violation zero probability. (In an extended version of this I discuss simple cases where introspectible, lexically conditioned factors do appear to affect the probability distribution.)

[63] This is not, recall, as simple a matter as saying that $\neg A$ is always an argument against A, and vice versa. (That case is of course also one where the relation is deductive.) And it takes a little bit of proving.

the False, or the empty set of worlds). Hence its introduction does not conform to well-established desiderata for explicit definition.

Let me add here a little excursion from the main line of argument. Non-conclusive argumentation also offers a useful perspective on intuitions underlying debate in philosophical logic. Consider distrust of Modus Ponens, a distrust motivated by its role in validating the eery, if not spooky *ex falso quodlibet*.[64] But *e.f.q.* is one of the two possible cases where entailment is not a special case of argumentative relevance as defined above: the **0**-proposition is universally irrelevant. Constraints proper to non-conclusive argumentation might thus play a normative role in restraining use of patterns of proof positive.[65]

If we pursue the main line of argument for the two most prominent binary connectives we find prominent conditions of use which make, respetively, for additivity (w.r.t. relevance) of 'and' and for convexity of 'or'. Both kinds of conditions rule out certain 'paradoxa of relevance' conceivable if nothing besides the probability axioms is assumed.

This raised the question of whether relevance and more generally, cardinal utilities) might not play a role in inducing semantic recursion analogous to the way in which truth or proof traditionally do. It turned out that this is not only conceivable, bu that observable and poorly defeasible constraints on their everyday use are explicable

- in terms of non-lattice-theoretic, indeed linear algebraic recursion induced by utility- or relevance-valuations;
- satisfying constraints on a Boolean semantic base that ensure satisfaction of intuitable desiderata spelt out in terms of non-conclusive argumentative relevance.

Condidering these constraints one has a suggestion of how utility- or relevance-structured semantic representations could induce truth- or proof-conditional ones.

In various ways, then, non-conclusive argumentative relations play a descriptive role in explaining what constrains our semantic intuitions. Philosophical logic must continue to rely on intuitions. in its therapeutic

[64] As Michael Dunn (1986:152) said: you don't want inconsistent information about the colour of your car let the Police Computer conclude that you are Public Enemy Number One.

[65] In return there are familiar deductive fallacies primed by inductive support, e.g. 'affirming the consequent' , i.e. concluding that $A \to B$ entails $B \to A$. Whereas if A is positively relevant to B we can conclude that B is positively relevant to A, never mind how much.

role it must also inveigh against them. But either way, the preliminary task is to investigate them, by any means possible.

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