A Semantics for Degree Questions Based on Intervals: Negative Islands and their Obviation

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Abstract

According to the standard analysis of degree questions (see, among others, Rullmann 1995 and Beck and Rullmann 1997), a degree question's LF contains a variable that ranges over individual *degrees* and is bound by the degree-question operator *how*. In contrast with this, we claim that the variable bound by the degree-question operator *how* does not range over individual degrees but over *intervals* of degrees, by analogy with Schwarzschild and Wilkinson's (2002) proposal regarding the semantics of comparative clauses. Not only does the interval-based semantics predict the existence of certain readings that are not predicted under the standard view, it is also able, together with other natural assumptions, to account for the sensitivity of degree questions to negative-islands, as well as for the fact, uncovered by Fox and Hackl (2007), that negative islands can be obviated by some properly placed modals. Like Fox and Hackl (2007), we characterize negative island effects as arising from the fact that the relevant question, due to its meaning alone, can never have a maximally informative answer. Contrary to Fox and Hackl (2007), however, we do not need to assume that scales are universally *dense*, nor that the notion of maximal informativity responsible for negative islands is blind to contextual parameters.

Introduction

The goal of this paper is to argue for a new approach to the semantics of degree questions. According to the standard analysis (see, among others, Rullmann 1995 and Beck and Rullmann 1997), a degree question's LF contains a variable that ranges over individual *degrees* and is bound by the degree-question operator *how*. In contrast with this, we will claim that the variable which is bound by the degree-question operator *how* does not range over individual degrees but over *intervals* of degrees, by analogy with Schwarzschild and Wilkinson's (2002) proposal regarding the semantics of comparative clauses.[1]

[1] Schwarzschild and Wilkinson (2002) briefly allude (p. 36) to the possibility of using a semantics based on intervals for the analysis of degree-questions, but they do not elaborate.

In other words, while, in the standard view, a sentence such as (1) corresponds to the (informal) logical form given in (2-a), we will assume that the right representation is the one given informally in (2-b):

(1) How fast is Jack driving?

(2) a. Standard View: for what degrees d of speed, is Jack driving d-fast?
b. Interval-Based Semantics: for what intervals I of degrees of speed, is Jack driving at a speed included in I?

As we'll argue, not only does the interval-based semantics predict the existence of certain readings that are not predicted under the standard view, it is also able (and this will be our main focus), together with other natural assumptions, to account for the sensitivity of degree questions to negative-islands, illustrated by (3) (cf. Rizzi 1990, Szabolcsi and Zwarts 1993, Rullmann 1995, among others). Further, we will show that this approach can also account for the fact, uncovered by Fox and Hackl (2007)[2], that negative islands can be obviated by some properly placed modals, as in (4) (we call this phenomenon *modal obviation*).

(3) a. How fast did Jack drive?
b. *How fast didn't Jack drive?
c. *How fast are we allowed not to drive?

(4) a. How fast are we not allowed to drive?
b. How fast are we required not to drive?

The relevant generalization, which is due to Fox and Hackl (2007), is the following:

> **Fox and Hackl's generalization**:
> Negative islands get obviated if negation immediately scopes either just above a possibility modal (as in (4-a)) or just below a necessity modal (as in (4-b)).

We will show that this generalization follows from the combination of the interval-based semantics and a principle, inspired by Dayal (1996) and Beck and Rullmann (1999), according to which any question presupposes that it has a maximally informative answer, i.e. a true answer that entails all the other true answers (what we call the *Maximal Informativity Principle*). Fox and Hackl's (2007) account of these facts, which we will present in the next section, also resorts to this principle, and in this respect our proposal is a direct heir to theirs; we will see, however, that our proposal allows us to dispense with two others of Fox and Hackl's (2007) assumptions, namely the hypothesis that every measurement scale is *dense* (see next section for a definition), and more importantly, the view that the presupposition induced by the Maximal Informativity Principle is computed in a way that is blind to contextual information (see subsection

[2]Similar facts had been already noted by Kuno and Takami (1997), but they did not recognize the differences among various modals.

1.2.3). Our approach will also be shown to make certain interesting predictions that previous approaches, as far as we can see, do not (cf. in particular sections 2.3, 3, 4.5).

1 Background

In this section we briefly review two previous approaches that are directly relevant to our own proposal. We start with presenting Rullmann's (1995) theory of negative degree islands in Section 1.1. Then we discuss its shortcomings, which lead us to the proposal in Fox and Hackl (2007) in Section 1.2.

1.1 Rullmann (1995)

Rullmann (1995), inspired by von Stechow's (1984) analysis of comparatives, offers an explanation for negative degree islands based on the following set of assumptions:

- The degree question operator *how* binds a variable ranging over degrees (in the scale determined by the adjective that *how* combines with):

  (5) Logical Form of *How tall is John?*:
  How_{d} [John is d-tall]?

- The interpretation of a degree question, similarly to Jacobson's (1995) theory of free relatives, involves a maximality operator, as shown below:

  (6) How tall is John?
  a. What is the maximal degree d such that John is d-tall?
  b. For what d, Max(λd.John is d-tall) = d

  (7) Max(D)= ιd.[d $\in$ D $\wedge$ $\forall$ d'$\in$ D [d' $\leq$ d] – *Max* is a partial function, as it is defined only if its argument has a maximal element.[3]

Given these assumptions, the oddness of negative degree questions can be explained as follows:

(8) *How tall isn't John?
a. What is the maximal degree d such that John is not d-tall?
b. For what d, Max(λd.John is not d-tall) = d

Let us assume that a lexical scalar predicate such as *tall* is *downward scalar*, i.e. licenses an inference from a degree d to any smaller degree d', as expressed by the following meaning postulate:

[3]Note that we intend the lambda-abstracts occurring in our semi-formal metalanguage to represent indifferently a function f from certain objects to truth-values or, derivatively, the set of objects whose characteristic function is f.

(9) $\forall x \forall d \forall d'$ (d' $\leq$ d) $\rightarrow$ (tall(x)(d) $\rightarrow$ tall(x)(d'))

Informally speaking, such a meaning postulate ensures that *being d-tall* is equivalent to *being d-tall or more.* It follows that the expression $\lambda d.$*John is not d-tall* denotes the set of degrees greater than John's height. This set however does not have a maximum, and therefore this expression will always be undefined.[4] This is how, according to Rullmann (1995), the negative island effect arises.

More generally, Rullmann (1995) predicts the following generalization:

(10) A question of the form How_d $\phi(d)$*?* (where ϕ is the complex degree-predicate created by wh-movement) is unacceptable whenever Max(λd.ϕ(d)) is undefined.

However, as was already noted in Beck and Rullmann (1997), this generalization is incorrect. For instance, it predicts the wrong outcome for questions such as the one below:

(11) How tall is it sufficient to be (in order to play basketball)?

A predicate ϕ of degrees is said to be upward scalar iff $\phi(d)$ entails $\phi(d')$ for any d, d' such that $d \leq d'$. It follows that in the case of upward scalar predicates there is no maximal degree that they are true of, and therefore the maximality operator cannot be applied to such a predicate. The degree predicate $\lambda d.$*it is sufficient to be d-tall* is upward-scalar, since if it is sufficient to be 7-feet tall or more, then it is sufficient to be 8-feet tall or more. Hence (11) is predicted to be unacceptable by Rullmann's (1995) theory, contrary to fact. Note furthermore that (11) is understood as asking for the *minimal* height such that it is sufficient to have that height. Thus even if the context could restrict the range of the degree variable in (11) so as to ensure that $\lambda d.$ *it is sufficient to be d-tall* does have a maximum, Rullmann's (1995) theory would still fail to predict the intuitive meaning of (11).

Another counterexample to (10) is clearly provided by all the cases which illustrate the phenomenon of *modal obviation*, e.g. cases where the degree-variable occurs under the scope of a negated possibility modal:

(12) a. How fast are we not allowed to drive?
b. For what speed d, are we not allowed to drive d-fast or more?

The degree variable in (12) occurs under the scope of a monotone-decreasing operator (negation), and the predicate $\lambda d.$*we are not allowed to be d-fast* does not have a maximum: for if we are not allowed to be d-fast or more, then we are not allowed either to be d'-fast or more, for any d' greater than d. Therefore, (12) is expected to be unacceptable - an incorrect prediction, as noted by Fox and Hackl (2007). Again, note that such a question is felt as asking for the

[4] Rullmann (1995) assumes an 'exactly' semantics for degree predicates. Since then an 'at least' semantics has become customary. On an 'exactly'-semantics, Rullmann's explanation for the negative island effect is that the expression $\lambda d.$*John is not d-tall* denotes the set of degrees that are either greater or smaller than John's height, and this set also has no maximum.

minimal degree of speed d such that we are not allowed to drive at a speed higher than or equal to d.

Beck and Rullmann's (1997) observation about cases such as (11) led them to replace Rullmann's (1995) concept of the maximal answer with that of the maximally informative answer, defined as a true answer that entails all the true answers. Now the meaning of a degree question can be informally paraphrased as below:

(13) What is the degree d that yields the most informative answer among the true propositions of the form $\phi(d)$?

This approach predicts the following generalization for degree questions:

(14) $\text{How}_{\text{d}}\ \phi(\text{d})$?

a. If ϕ is *downward-scalar* (i.e. $\phi(d)$ entails $\phi(d')$ whenever $d' \leq d$), the question asks for the highest degree d such that $\phi(d)$ is true.

b. If ϕ is *upward-scalar* (i.e. $\phi(d)$ entails $\phi(d')$ whenever $d \leq d'$), the question asks for the smallest degree d such that $\phi(d)$ is true.

This predicts the right interpretation for (11): this is so because now the question asks for the *smallest* degree sufficient to be a basketball player. However, as the authors themselves notice, while Beck and Rullmann's (1997) account predicts the right results for the data in (11) (and also for the basic cases of modal obviation, which they did not discuss), now the basic explanation for negative-island effects is lost:

(15) How tall isn't John?

a. For what d, John isn't d-tall

b. For what d, John is less than d-tall

Suppose that John's height is just below 6-feet. Then the set of true answers is:

(16) {John isn't 6-feet tall, John isn't 6.5-feet tall, John isn't 7-feet tall, ...}

Clearly 'John isn't 6-feet tall' is the maximally informative answer. And yet the question is unacceptable.

1.2 Fox and Hackl's (2007) account of negative islands and of modal obviation

To remedy the above problem and to explain the paradigm in (3) and (4), Fox and Hackl (2007) maintain Beck and Rullmann's (1997) view that degree-questions (and questions in general) ask for a maximally informative answer, but add a specific assumption regarding the structure of scales – what they call *the universal density of measurement scales*. More specifically, they make the following assumptions:

- Universal density of measurement: all scales are *dense*, i.e. for any two degrees d_1 and d_2 in a given scale, there is a degree d_3 between d_1 and d_2:

$$\forall d_1 \forall d_2 \ ((d_1 < d_2) \rightarrow (\exists d_3 \ d_1 < d_3 < d_2))$$

- Maximal Informativity Principle (inspired by Dayal 1996 and Beck and Rullmann 1999, among others): any question presupposes that it has a maximally informative answer, i.e. a true answer which logically entails all the other true answers.

1.2.1 Simple negative islands

Given the above assumptions, the logical form for a question like (3-b) is the one given in (17), and the presupposition induced by the Maximal Informativity Principle amounts to the claim that among all the true statements of the form *Jack did not drive d-fast or more than d-fast*, there is one that entails all the others.

(17) How_d [Jack did not drive d-fast]?

Note that for any d, d', with d $\leq$ d', the proposition that Jack didn't drive d-fast entails the proposition that Jack didn't drive d' fast (for if d $\leq$ d', then the proposition that Jack's speed is at most d entails the proposition that Jack's speed is at most d'). So the maximally informative true answer, if it exists, must be based on the *smallest* degree d such that Jack's speed was not d or more than d. Now suppose that Jack's exact speed was 50mph. Then for any d > 50, Jack did not drive d-fast; but is there a *smallest* d such that Jack did not drive d-fast, i.e. a smallest d above 50? Assuming that the scale of speeds is dense, there cannot be such a degree: for any degree $50 + \epsilon$, however small ϵ is, there is another degree $50 + \epsilon'$ strictly between 50 and $50 + \epsilon$ (take ϵ' smaller than ϵ). Therefore the presupposition that there is a maximally informative true answer can never be met.

1.2.2 Modal obviation

The acceptability of the questions in (4) is accounted for as follows. (4-a)'s logical form is (informally) the following:

(18) For what degree d of speed, we are not allowed to drive d-fast?

Now suppose that the law states that we are not allowed to drive at 65mph or faster, and says nothing more. Then for any speed d strictly smaller than 65mph, it is false that we are not allowed to drive d-fast. Hence 65mph is the smallest speed d such that we are not allowed to drive d-fast or more; furthermore, not being allowed to drive at 65mph or more entails not being allowed to drive at any higher speed. Hence the proposition that we are not allowed to drive at 65mph or more is the most informative true proposition of the form *we are not allowed to drive d-fast*, and the Maximal Informativity Principle is satisfied. In other words, the condition that there be a maximally informative answer can be met in the case of (4), which is why such a question is acceptable.

This reasoning extends trivially to (4-b), simply because *being required not to do X* is equivalent to *not being allowed to do X*. Fox and Hackl (2007) also show that in cases where negation *follows* a possibility modal, as in (3-c), or precedes a necessity modal, the corresponding questions are predicted to be unacceptable.

1.2.3 Apparently discrete scales, modularity, and blindness to contextual information

Fox and Hackl's (2007) account relies in a crucial way on the assumption that scales are dense. This is certainly natural when we talk about speed or height. But, as Fox and Hackl themselves point out, there are contexts in which we don't treat even such scales as dense: for instance, when we say that Jack and Sue are as tall as each other, we do so relatively to a certain standard of *granularity*, so that Jack might be, say, half a millimeter smaller than Sue, and yet be said to be as tall as her, because such a tiny difference falls below that standard. In a context in which heights are rounded, say, to centimeters, the relevant scale is not dense; but then if Jack's height is 180cm, the smallest height h such that he is not h-tall exists, and is simply 181cm (because intermediate values are not relevant). So in a context of this sort, a question like *How tall isn't Jack?* should have a maximally informative answer, and therefore negative islands should be obviated most of the time (since virtually every context is of this sort). Fox and Hackl's (2007) approach avoids this potential problem in the following way: for the purpose of deciding whether a degree question is grammatical or not, grammar makes use of a purely logical notion of entailment, as opposed to a contextual notion of entailment which could be relativized to various contextual parameters, e.g. to a granularity parameter. More specifically, Fox and Hackl (2007) assume that the language faculty includes a modular, encapsulated *Deductive System* (DS), which, among other things, checks whether the presupposition induced by the Maximal Informativity Principle for a given question is or is not contradictory, on the basis of a purely logical notion of entailment. If the relevant question passes this test (i.e. has a non-contradictory presupposition from the point of view of DS), it can then be interpreted in a real situation, and various contextual parameters are eventually factored in.[5]

The importance of the assumption that all scales are dense is particularly clear when one turns to scales that are not intuitively dense. Consider for instance the following contrast:

(19) a. How many children does John have?
b. *How many children doesn't John have?

The unacceptability of (19-b) is explained in exactly the same way as that of (3-b). Suppose John has exactly three children. Then John, for sure, does not

[5]We will return to this aspect of Fox and Hackl's (2007) modularity thesis in section 3.

have four children; but it is also true that he does not have 3.5 children, or 3.05 children, or 3.005 children, and so on ... Hence there does not exist a minimal number n such that Jack does not have n children or more, and therefore (19-b) can not have a maximally informative answer. But if the degree variable in (19-b) could be restricted to range over natural numbers, there would actually exist a minimal degree n such that Jack does not have at least n children - namely, take n = 4. But the universal density of measurement scales entails that there is no scale consisting of just the natural numbers.

Such a view can be defended on the ground that the knowledge that the number of children someone has is an integer is not purely linguistic or "logical"; rather, it is a form of lexical or encyclopedic knowledge, and therefore there is no reason why *how many* should be constrained to bind a variable whose range of values includes only the natural numbers. Yet in normal contexts, the *relevant* values for the variable bound by *how many* in (19-b) consist only of the integers; if it were possible to restrict the range of the numerical variable to a salient domain of quantification (such as the natural numbers), the condition that there be a maximally informative answer would actually be met for (19-b). So Fox and Hackl, again, do not only need to claim that all scales are dense; they also need to assume a modular system in which some semantic and pragmatic processes operate in isolation and are blind to contextual information, in particular to possible contextual restrictions on the range of variables. As already mentioned, the *Deductive System*, which in Fox and Hackl (2007) is responsible for negative islands, is assumed to be encapsulated from these contextual factors.

Fox and Hackl's (2007) explicit goal is to challenge certain widely accepted assumptions regarding the relationship between grammar, pragmatic processes, lexical meaning and contextual factors. We consider it worthwhile to investigate an alternative account of the negative island facts which can dispense with the hypothesis that grammar always treats all scales as dense, and thus does not need to assume that the mechanism responsible for negative islands cannot interact with contextual parameters. We should note, however, that Fox and Hackl (2007) also argue that the universal density of measurement scales is able to account for many other interesting facts outside the realm of degree questions, facts which we do not address in this paper. We do not provide here a complete comparison between our approach and that of Fox and Hackl (2007), which would have to wait for the future.

2 The Interval-Based Account

In this section, we present a first version of our proposal (subsection 2.1), and then show how it accounts for the basic paradigm described in (3) and (4) (subsection 2.2). Then we will provide some independent motivation for the proposal (subsection 2.3). Finally, in subsection 2.4, we will briefly compare our proposal to previous ones, including that of Szabolcsi and Zwarts (1993).

2.1 The proposal

Our proposal is based on the following two ingredients:

- Instead of treating scalar predicates as denoting a relation between individuals and degrees, we assume that they denote a relation between individuals and *intervals* of degrees[6] (as argued by Schwarzschild and Wilkinson 2002[7]):

  (20) $\|\text{tall}\| = \lambda D_{<d,t>}$: D is an **interval**. λx.x's height $\in$ D (where *interval* is defined as in (21))

  (21) Given a scale E, i.e. an ordered set (E, $\leq$), an *interval* on E is a subset D of E such that:
  $\forall d_1 \in E\ \forall d_2 \in E\ \forall d_3 \in E\ (d_1 \in D \wedge d_3 \in D \wedge d_1 \leq d_2 \leq d_3) \rightarrow d_2 \in D$

  Note that the notion of interval is well defined for discrete scales as well as for dense scales.

  As a result, the LF of a *how tall* question will be (22-a), which can be informally paraphrased as in (22-b):

  (22) a. $\text{How}_{D_{<d,t>}}$ [... D-tall ...]
  b. For what intervals D of degrees of heights, is it the case that [... D-tall ...]?

- Like Fox and Hackl (2007), we adopt a version of the *Maximal Informativity Principle*, with a slight modification: we view the principle as requiring not only that in every world compatible with the common ground, there be a true answer that entails all the true answers, but also that this maximally informative answer not be the same in every world of the common ground. In other words, it must be common knowledge that there is a maximally informative answer, but it must not be common knowledge what this maximally informative answer is.

  We use the term *answer* in a narrow, technical sense: an *answer* to a degree-question of the form *[How*$_D$ $\phi(D)$*]* is a proposition that belongs to the Hamblin-set of the question, i.e. a proposition that can be expressed as $\phi(D)$, for some interval D. Crucially, many propositions that

[6] See however Section 4, in which we modify this basic proposal.

[7] As Schwarzschild and Wilkinson (2002) notice, the idea to use intervals in the semantic analysis of degree constructions has been proposed before them, but they rightly point out that these previous works (Seuren 1984; Bierwisch 1989; Kennedy 1999, 2001) are basically "extensions of degree approaches". For instance, while Kennedy (2001) *construes* degrees as open intervals on a scale, he does not use intervals of degrees (which in his system would be higher-order intervals whose 'points' are themselves intervals, given that degrees are defined as intervals).

can intuitively serve as answers are not in this set.[8]

Here is a formally more explicit version of the Principle:

(23) a. **Definition**: An answer A to a question Q is a *Maximally Informative Answer to* Q *in a world* w if A is true in w and entails all the answers to Q that are true in w.[9]

b. **Maximal Informativity Principle (MIP).**
A question Q presupposes that for every world w compatible with common knowledge, there is an answer A to Q such that:
(i) A is the maximally informative answer to Q in w.
(ii) For at least one other world w' compatible with common knowledge, A is not the maximally informative informative answer in w'.

In the rest of this paper, we will sometimes say that the Maximal Informativity Principle is or is not satisfied for a certain question in a certain situation, or in a certain world; properly speaking, however, the Maximal Informativity Principle can only be satisfied by a context, viewed as a set of worlds. When we say that the Maximal Informativity Principle is satisfied in a certain situation (or world), we simply mean that there is an answer to the question which is true in that situation (or world) and which entails all the other answers that are true in this situation (or world).

As in Fox and Hackl (2007) and Abrusán (2007), weak islands configuration will be characterized as configurations which give rise to questions that can never have a maximally informative answer, i.e. for which the presupposition induced by the Maximal Informativity Principle is a logical contradiction.

2.2 Explaining the basic paradigm

2.2.1 Simple degree questions

A simple degree question such as (24-a) receives the interpretation given in (24-b).

(24) a. How tall is Mary?
b. For what interval I, does Mary's height belong to I?

[8] As far as we know, every semantic account of weak islands, starting with Szabolcsi and Zwarts (1993), needs to make very specific assumptions about the structure of the set of possible answers to a given question. See Section 2.4 for a short comparison of our approach with that of Szabolcsi and Zwarts (1993). Note that if the set of *answers*, in the narrow, technical sense, were such that the grand conjunction of any set of answers - possibly infinite - were itself an answer, there would always be a maximally informative answer, since the grand conjunction of all the true answers would then itself be an answer that entails all the true answers.

[9] Note that in a given world w, either there is a *unique* maximally informative answer, or there is none.

Now let h be Mary's height. Clearly, any answer based on an interval that includes h is a true answer(where an answer is said to be 'based' on an interval I if it expresses the proposition that Mary's speed was not in I); furthermore, the proposition that Mary's height belongs to a given interval I_1 entails the proposition that Mary's height belongs to I_2, for any I_2 that includes I_1. Consequently, the proposition that Mary's height belongs to the interval *[h,h]* (i.e. *is h*) expresses a true answer that entails all the other true answers, hence is the maximally informative answer.[10] A fully cooperative and informed speaker is expected to use this maximally informative answer, i.e. to give Mary's exact height as an answer - clearly a correct prediction. So in every possible world, there is a true answer that entails every true answer and therefore the Maximal Informativity Principle is met as soon as this answer is informative, i.e. as soon as the context does not already entail it.

2.2.2 Predicting Negative Islands

We now turn to negative islands. According to the interval-based semantics, (25-a) is interpreted as (25-b).

(25) a. # How fast didn't Mary drive?
     b. For what interval I, Mary's speed was not in I

The unacceptability of (25) is derived in two steps:

- First, let us show that (25) has a maximally informative answer if and only if Mary's speed was 0. Let s be Mary's speed. Let us assume that s is distinct from 0. Then the set of intervals such that s is not in them consists of a) all the intervals strictly above s, and b), all the intervals strictly below d. Hence the set of all the true answers to (25) is the set of answers based on such intervals. Now note that any answer based on an interval above s fails to entail an answer based on an interval strictly below s, and vice versa. Therefore there cannot be a true answer to (25) that entails all the other true answers. To sum up, if s is distinct from 0, there cannot be a maximally informative answer. If s = 0, then there is a true answer that entails all the true answers, namely, the proposition that states that Mary's speed was not in the interval $]0, +\infty[$, which is equivalent to the claim that s = 0. So (25) has a true answer that entails all the true answers if and only if Mary's speed was 0.

- Second, let us temporarily assume that the Maximal Informativity Principle is satisfied in a certain context. Then in every world compatible with common knowledge, there is a true answer that entails all the true answers, i.e., given what has just been shown, in every such world Mary's

[10] Throughout the paper, we employ the ISO notation for intervals, where inwards pointing brackets indicate the inclusion of the endpoint, while outward pointing brackets indicate the exclusion of the endpoint.

speed was 0. This means that (25) can be felicitous only when it is common knowledge that Mary's speed was 0, (i.e., equivalently, that her speed was not included in $]0, +\infty[$). But then the maximally informative answer, namely the proposition that Mary's speed is not included in $]0, +\infty[$, is in fact already entailed by the common ground and necessarily known to be the maximally informative answer. So the second clause of the Maximal Informativity Principle ((23-b-ii)) is not satisfied.

It follows that the Maximal Informativity Principle can never be satisfied. Note that the above proof does not make reference to whether the underlying scale is dense or discrete, a point that we will emphasize in subsection 2.2.5.

2.2.3 Accounting for Fox and Hackl's (2007) obviation facts

Recall that a possibility modal scoping below negation is able to obviate negative islands, as was illustrated by example (4-a), repeated here as (26):

(26) How fast are we not allowed to drive?

According to the interval based account, (26) receives the following interpretation:

(27) For what interval I, it is not allowed that our speed be in I?

Now suppose that the law determines a maximal permitted speed and does nothing more. Call this speed limit l. Then the set of all the intervals I such that it is not allowed that our speed be in I is the set of all the intervals that are strictly above l. Consider in particular the answer to (26) based on the interval $]l, +\infty[$. Clearly, this answer is true and entails all the other true answers (since the information that our speed must not be in $]l, +\infty[$ tells us everything there is to know in this context). Therefore, the Maximal Informativity Principle is satisfied in every context in which it is common knowledge that the law determines a maximal permitted speed and does not impose any other regulation regarding speed, but it is not yet known what this speed limit is – otherwise the second clause of the Maximal Informativity Principle ((23-b-ii)) would not be satisfied.[11]

This reasoning extends straightforwardly to the case where negation scopes just below a necessity modal, as in (4-b), because being required not to drive at a speed contained in a given interval I is equivalent to not being allowed to

[11] Consider a context in which it is already common knowledge that the speed limit is either 70mph or 80mph, and nothing more is known. In such a context, the proposition that we are not allowed to drive above 80mph is true in every world compatible with common knowledge, but it is not the maximally informative answer to (26) in every such world, since in worlds where the speed limit is 70mph, the maximally informative answer is the proposition that we are not allowed to drive above 70mph. So the Maximal Informativity Principle is satisfied in such a context (its second clause (23-b-ii), in particular). Thanks to an anonymous reviewer for drawing our attention to this case, which led us to improve our formulation of the MIP.

drive at a speed contained in I.[12]

2.2.4 No obviation if the possibility modal scopes above negation

When the possibility modal takes scope not below but above negation, then the question is not acceptable, as was shown in (3-c), which we repeat below in (28-a), and whose interpretation according to the interval-based semantics is given in (28-b):

(28) a. # How fast are we allowed not to drive?
 b. For what interval I, it is allowed that our speed not be in I?

To show how this prediction comes about, let us first point out the following logical fact:

(29) For any two intervals I_1 and I_2, the two following statements are equivalent:
 a. *It is allowed that our speed not be in I_1* entails *It is allowed that our speed not be in I_2*
 b. I_2 is included in I_1

From the observation in (29), it follows that the maximally informative true answer to (28), if it exists, is based on an interval that includes all the intervals that yield a true answer. Let us call this interval the 'maximally informative interval'. Let us then consider several possible cases:

- First case. There is no particular speed s such that our speed must be exactly s.

  In this case, for any speed d, we are allowed not to drive at speed d, i.e. it is permitted that our speed not be in the (degenerate) interval $[d, d]$. But given that the maximally informative interval M, if it exists, must include all the intervals I such that it is allowed that our speed not be in I, M must in fact be $[0, +\infty[$. But this would mean that it is allowed that our speed not be in $[0, +\infty[$, i.e. that there is a permissible world in which we have no speed (not even the null speed), which is obviously contradictory. Hence there cannot be a maximally informative interval.

[12] As was pointed out to us by Danny Fox (p.c.), our interval-based semantics predicts, contrary to Fox and Hackl's (2007) proposal, that (26) ('How fast are not allowed to drive?') can also be felicitous in a context in which it is known that there is a *minimal* required speed and no maximal permitted speed, which seems to go against the intuitions of some speakers. Nevertheless, several native English speakers have no problem finding questions of this form felicitous even in a context of this type, especially when we move to a case where the absence of a maximal permitted degree is more plausible. For instance, if it is known that a certain swimming pool is forbidden to children under a certain age, and no other rule exists, one could felicitously ask the structurally parallel question 'How old can't a child be if he is in the pool?'. More investigation is thus needed. We refer the reader to Abrusán and Spector (2008) for a potential (stipulative) solution. In subsection 4.5 we discuss a somewhat related issue.

- Second case. There is a particular speed s, distinct from 0, such that our speed must be exactly s.

  Since s $\neq$ 0, it is allowed that our speed not be 0, and it is also allowed that our speed not be d, for any d strictly above s. Hence the maximally informative interval, if it exists, must include both 0 and any speed above s, hence must again be $[0, +\infty[$, which, as explained above, is contradictory.

- Remaining case. Our speed must be exactly 0.

  It follows from the two previous reasonings that there can be a true answer that entails all the true answers only if our speed must be exactly 0. In such a case there is a true answer to (28) that entails all the true answers, namely the proposition according to which we are allowed not to drive at a speed included in $]0, +\infty[$.

We can now conclude that the Maximal Informativity Principle can never be satisfied for (28-a). For suppose that it were satisfied in a certain context C. Then, in every world of C, there is a true answer that entails all the true answers. As shown above, this is equivalent to saying that in every world of C, it is required that our speed be 0, i.e. that in every world, the maximally informative answer is the one based on the interval $]0, +\infty[$. But then the second clause of the Maximal Informativity Principle ((23-b-ii)) is in fact not satisfied in C, because the maximally informative answer is the same in all the worlds of C.

2.2.5 Discrete scales

Our proposal does not rely on any particular assumption about the precise structure of scales, besides the fact that they are totally ordered sets. So the account works equally well for dense and discrete scales. Let us have a brief look at negative *how many*-questions.[13]

(30) a. # How many children doesn't Jack have?
     b. For what interval I of integers, the number of children that Jack has is not in I?

The explanation of the deviance of (30-a) is exactly the same as in the case of (25-a). Namely, if Jack has, say, exactly n children, then for any interval I either strictly above or stricly below n, it is true that the number of children that Jack has is not in I. But for any I_1 strictly above n and any I_2 stricly below n, neither the answer based on I_1 entails the one based on I_2, nor does

[13] As is well known, negative islands in degree questions arise only for one type of reading, sometimes called the *amount*-reading, which is usually analyzed as involving reconstruction of the numerical variable; see Heycock (1995) and Fox (1999) and the references cited therein. A question such as (30-a) strongly favors this reading, because the other, non-reconstructed reading, which is not subject to negative islands, would be paraphrased as "For what number n, are there n children such that Jack does not have them?", which does not make sense in normal contexts.

the latter entail the former. Therefore the only case where there could be a maximally informative answer is when $n = 0$. But then any context in which it is known that there is an answer that entails all the true answers is in fact a context in which it is already known that Jack has no children, i.e. in which the maximally informative answer is the same in every world compatible with common knowledge. So the Maximal Informativity Principle can never be met.

2.3 The interval-based reading exists

Having shown that the combination of the interval-based semantics and the principle of maximal informativeness accounts for the basic facts of negative islands as well as Fox and Hackl's generalization, we now point out independent evidence in favor of the interval-based reading in the case of embedded interrogatives. Further evidence will be adduced in section 3.

Consider the following question:

(31) How fast must one drive on this highway?

Suppose that in actual fact, it is required to drive at a speed between 45mph and 75mph on the relevant highway. According to Beck and Rullmann's (1997) and Fox and Hackl's (2007) view of degree-questions, the complete answer to (31) should be the most informative proposition of the form *one must drive at Xmph at least*, i.e., in this situation *One must drive at 45mph at least*. According to the interval-based semantics, the complete answer in this situation is strictly stronger, as it is the proposition that states that one must drive between 45mph and 75mph. Most informants agree that both answers could be given by a fully informed and cooperative speaker. This suggests that both the readings predicted by the standard account and by the interval-based semantics exist. On the one hand, the existence of the second reading provides support for our proposal; on the other hand, we do not account for the possibility of another reading corresponding to what is predicted by the 'standard' approach. In Section 4 we present a more sophisticated version of our analysis that is able to predict both readings while still accounting for the facts uncovered by Fox and Hackl (2007). For the purposes of this section, our aim is to establish the existence of the interval-based reading.

We can confirm the existence of the interval-based reading by paying attention to the interpretation of *embedded* degree questions. Thus consider the interpretation of (32-b) below, uttered just after the discourse given in (32-a):

(32) a. Jack and Peter are devising the perfect Republic. They argue about speed limits on highways. Jack believes that people should be required to drive at a speed between 50mph and 70mph. Peter believes that they should be required to drive at a speed between 50mph and 80mph. Therefore ...

b. Jack and Peter do not agree on how fast people should be required to drive on highways.

The point is that (32-b) can clearly be judged true in the context given in (32-a). Let us assume (following for instance remarks by Sharvit 2002) that for X and Y to disagree on a given question Q, there must be at least one potential answer A to Q such that X and Y do not assign the same truth-value to A (in the technical and narrow sense of the term 'answer', cf. subsection 2.1). Then for Jack and Peter to disagree on how fast people should be required to drive, i.e. for (32-b) to be true, there must be at least one answer to *How fast should people be required to drive on highways?* about which Jack and Peter disagree. But note that in the context described in (32-a), Jack and Peter do not actually disagree about the minimal permitted speed; in other words, they agree on the truth value of every proposition of the form *People should be required to drive at least d-fast*. So according to the 'standard' view of degree questions, (32-b) is false in such a context (since according to this view, the potential answers to the embedded degree questions in (32-b) are precisely the propositions of the from *People should be required to drive at least d-fast*, about which Jack and Peter have no disagreement). Whether or not there is a reading according to which (32-b) is false given (32-a), it is clearly possible to interpret it as true in this scenario. Therefore the standard view is insufficient. Furthermore, the interval-based analysis straightforwardly accounts for this truth-value judgment. According to the interval-based semantics, (32-b) means that for at least one interval of speeds I, Jack and Peter do not agree on the truth-value of *People should be required to drive at a speed included in I.* And this is clearly the case in the scenario given in (32-a) - namely, Jack believes that people should be required to drive at a speed contained in $I = [50, 70]$, while Peter thinks this is not so.

In section 4.5, we will discuss other cases which provide direct evidence that the interval-based reading exists.

2.4 Comparison with previous approaches

Our proposal belongs to a family of proposals whose general logic is analyzed in a recent paper by D. Fox (Fox 2007b): certain types of failures arise when no candidate within a set of alternatives can count as the most informative answer, so that no such candidate can receive an 'exhaustive' reading; exhaustification becomes however possible when the relevant alternatives are embedded under a necessity modal – cf. Fox's (2007b) statement #46. Fox (2007b) suggests that whenever we observe a similar pattern (in which a certain type of failure appears to be obviated within the scope of necessity modals), it is a good strategy to approach it in these terms. Besides the works already cited, see also Abrusán (2008), among others, which applies this logic to manner questions.

These types of proposals all rely on specific assumptions about the structure of the domain associated with the variable bound by the relevant wh-operator - in our case the assumption that the variable bound by a degree question operator ranges over intervals, in Fox & Hackl's case the the assumption that degree-variables range over a *dense* scale. In this respect, these proposals develop an intuition that was already present in Szabolcsi and Zwarts (1993). According to Szabolcsi and Zwarts (1993), negative islands arise when the *wh*-word denotes in a domain on which the Boolean operation associated with negation, namely complementation, is not always defined. They propose that amounts and numbers denote in structures that are (non-free) join semilattices and lattices, respectively. As these structures are not closed under complementation, negation is predicted to lead out of these structures in some cases. Within our own proposal, the fact that there is no maximally informative answer to simple negative degree questions could also be expressed in algebraic terms, by saying that complementation is not always defined on this domain, the domain of intervals. That is, complementation can take us out of the domain of intervals; in other words, the complement of an interval is generally not an interval itself. In this respect, our proposal, as well as Fox and Hackl (2007), builds on Szabolcsi and Zwarts's (1993) fundamental intuition that weak islands have to be accounted for in terms of the algebraic properties of the relevant domains.

Yet there are both empirical and conceptual differences between proposals based on the Maximal Informativity Principle, such as ours and that of Fox and Hackl (2007), and that of Szabolcsi and Zwarts (1993). At the empirical level, Szabolcsi and Zwarts's (1993) paper is designed to match the predictions of syntactic approaches to weak-islands based on Relativized Minimality (see Rizzi 1990 and the references cited therein); as a result, Szabolcsi and Zwarts (1993) do not account for the phenomenon of modal obviation. There is also an important difference at the conceptual level. Szabolcsi and Zwarts (1993) postulate that as soon as a certain operation is not always defined in a certain domain, a question whose interpretation involves this operation in this domain is unacceptable. In our proposal, as well as in Fox and Hackl's (2007), the role played by the algebraic properties of the domain of degrees is much more indirect: negative islands arise when the Maximal Informativity Principle can be satisfied in no context, a state of affair which comes about because of the algebraic properties of the domain. Furthermore, while for Szabolcsi and Zwarts (1993) negative islands arise because for *some* values of the degree variable the complement is not defined, in the present proposal, simple negative degree questions are unacceptable due to the fact that a complement is *never* defined, to the effect that the presupposition induced by the Maximal Informativity Principle is contradictory. As we'll see in section 3, we predict that in some other cases the presupposition induced by the Maximal Informativity Principle is not contradictory, but imposes very stringent constraints on the contexts in which the relevant question is appropriate.[14] This type of context sensitivity will be shown to be a welcome property of our system.

3 Further Predictions: Quasi negative islands

In this section, we discuss cases which our account predicts to be acceptable only in very specific, quite implausible contexts, to the effect that the relevant

[14] In this respect our proposal differs significantly from that of Fox and Hackl (2007), as explained in section 3.

questions sound deviant out of the blue, but can be rescued if evaluated with respect to an appropriate context.[15] We call this class of examples 'quasi negative islands'.

3.1 Intervention of Negative Universal Quantifiers

Consider the following degree questions:

(33) How many children does none of these women have?

Out of the blue, (33) is felt as odd, and does not sound significantly better than a simple negative degree question. There are however contexts, though admittedly not very plausible ones, which make (33) felicitous: suppose that you know that none of the relevant women has ten children or more, but that for any number smaller than ten, at least one of the relevant women has exactly that number of children. In other words, at least one of them has exactly one child, while another one has exactly two, and so on up to nine. In such a context, the question would make sense and you should answer it with 'ten', or 'ten or more', or something equivalent. It turns out that the interval-based semantics predicts that (33) can be felicitous only in a context of this sort; more specifically, (33) is predicted to presuppose that there is a number n such that none of the relevant women has n children or more but for every m below n, some woman has exactly m children. Let us see how this prediction comes about.

Observe first that the LF for (33) is as in (34), which is equivalent to (35)[16]:

(34) How_{I} [[none of these women] [λx.[λI. x's number of children is ∈ I]]]

(35) For which interval I, no woman x among these women is such that x's number of children is included in I?

Now, let us point out that the two statements in (36) are equivalent, for any two intervals of numbers I_1 and I_2:

(36) a. *No woman x is such that x's number of children is in* I_1 entails *No woman x is such that x's number of children is in* I_2
b. I_2 is included in I_1

It follows that if there is a maximally informative answer to (33), it is 'based' on an interval that contains all the other intervals that would give rise to a true answer. Any answer based on an interval which is not included in this 'maximal' interval has to be false. So the Maximal Informativity Principle, applied to (33), yields the following presupposition:

[15]Similar effects have been noted in Kroch (1989), who has argued that negative amount questions improve if the context entails the existence of a unique salient amount that could be the answer of a question. Kroch (1989) however gives no explanation of these facts.

[16]Note that this prediction will not be affected the modification we add to our proposal in section 4, as will be explained in section 4.3.

(37) There is an interval of numbers I such that none of the relevant women has a number of children included in I, and for any interval I' not included in I, at least one of the relevant women has a number of children included in I'.

(37) is in turn equivalent to:

(38) a. There is an interval of numbers I such that none of the relevant women has a number of children included in I, and for any number not included in I, at least one of the relevant women has that number of children.

b. There is an interval I such that for every number except those in I, one of the relevant women has that number of children

Now, suppose that (38), i.e. that there is an interval I such that for every number except those in I, some woman (in the relevant group of women) has that number of children. In principle, I could be either of the form $[0, m]$, $[n, m]$ or $[n, +\infty[$. In the former two cases, it would follow that for any number above m, some woman in the denotation of *these women* has that number of children, hence that there are infinitely many women and infinitely many children.[17] Given that the denotation of *these women* can be assumed to contain finitely many women, we are left with the case where I is of the form $[n, +\infty[$, and the predicted presupposition finally amounts to the following:

(39) For some number n, no woman has n children or more, and for every m such that $m < n$, some woman has exactly m children.

This prediction seems to us to be at least a good approximation of the observed facts: a question such as (33) clearly sounds odd out of the blue, and good in contexts in which (39) is entailed by the common ground. Note that if it is known that all the relevant women have exactly the same number of children, (33) clearly sounds quite odd.[18] Likewise, if it were known that half of the women have exactly 6 children and that the other half all have an identical number of children different from 6, the question is again infelicitous.[19]

[17]Unless there were some clear contextual restriction on the range of the degree variable.We do not consider this possibility here, but as far as we can see, our general point – i.e. that we correctly predict that sentences such as (33) trigger very stringent presuppositions that are not predicted by competing proposals – would not be affected if we did.

[18]Fox (p.c.) suggests that this fact might be completely independent of degree questions: he points out that a wh-question such as 'Which books did none of these women read?' suggests, among other things, that the ten women in question did not all read the same books.

[19]Our prediction is however only approximately correct, because (33) is felicitous also in some contexts which are in some intuitive sense 'close' to the situation described in (39) but in which (39) is, strictly speaking, false. For instance, (33) sounds felicitous even in cases where, say, it is already known that every woman has at least two children, and that for every number m up to a certain number n, some woman has exactly m children - and in this context, it is not the case that for every n smaller than a certain m, some woman has exactly n children, since no woman has exactly one child. Section 4.4 will introduce some machinery that is able to address this problem.

It is worth pointing out that previous proposals do not make the same predictions as ours. Rullmann (1995) and Szabolcsi and Zwarts (1993) predict (33) to be unacceptable for the same reasons as simple negative degree questions are, i.e. because these proposals rule out every negative degree questions. The situation is more complicated in the case of Fox and Hackl (2007). They predict (33) to be acceptable and to presuppose very little (if anything at all), but the reasoning leading to this conclusion is quite complex.

According to Fox and Hackl (2007), (33) receives the following (informal) logical form:

(40) For which n, none of these women has n children or more?

For the presupposition induced by the Maximal Informativity Principle to be satisfied, there must be a *minimal* number n such that none of the relevant women has n children or more, (minimal in the sense that for any number m below n, at least one woman has m children or more). Now, suppose that there are finitely-many women in the denotation of *these women*, as would be the assumed by speakers and addressees in any realistic context. Consider the woman who has the greatest number of children, and let m be that number. Then set of numbers k such that no woman has k children or more is the interval $]m, +\infty[$. Given that the scale of numbers is dense (by hypothesis), this set does not have a minimal member, and therefore the Maximal Informativity Principle is not satisfied. So it could seem that Fox and Hackl (2007) predict (33) to be generally unacceptable. But this is in fact not so, because the assumption that that the set of women referred to by *these women* is finite is by no means a logical necessity; in fact, as we'll explain shortly, the Maximal Informativity Principle is met in certain worlds in which this set is infinite.

The general picture Fox and Hackl (2007) offer is the following: for a degree question to satisfy the Maximal Informativity Principle, it is only required that what they call the *Deductive System* (DS)[20] be incapable of deriving a contradiction from the assumption that the Maximal Informativity Principle is satisfied. DS is a component of the grammar which a) is blind to contextual information, and b) operates on impoverished logical forms.[21] As a result, DS does not have access to the information that the relevant domain of quantification (i.e. the set of women referred to by the demonstrative *these women*) is finite. Now, there are worlds in which the Maximal Informativity Principle is satisfied for (33), and in those worlds the reference of *these women* includes infinitely-many women. More precisely, once the universal density of measurement is taken into account, the Maximal Informativity Principle is satisfied in exactly all the worlds in which there is a number m such that none of the relevant women has m children or more, and such that there is an interval of numbers $[k, m[$ such that for every value x in $[k, m[$ (and there are infinitely-many such values be-

[20] Cf. section 1.2.3 of the present paper and Fox and Hackl's (2007) section 5.

[21] Fox and Hackl's (2007) characterization of DS is directly inspired by a paper by Jon Gajewski (Gajewski 2002); DS ignores everything except the sentence's syntactic structure and the meaning of the 'logical' vocabulary.

cause the relevant scale is dense), at least one woman has exactly x children. So (33) is predicted to be acceptable in the sense that the Maximal Informativity Principle does not lead DS to a contradiction (in other words, the presupposition induced by the Maximal Informativity Principle is not contradictory for DS).

But this is only a first step in Fox and Hackl's (2007) interpretative procedure.[22] Since the actual interpretation of sentences is known to take into account contextual information (such as domain restrictions and granularity parameters), Fox and Hackl (2007) allow a question that has passed the test of the Maximal Informativity Principle from the point of view of DS, and is thus acceptable, to receive a 'realistic interpretation' in which contextual information is taken into account. So (33), once determined to be grammatical, will end up being interpreted relatively to a scale that only includes integers, and the restriction of the negative quantifier to a finite domain will also be factored in. The resulting interpretation is then the following: (33) is predicted to ask for the smallest *integer* n such that no women has n children or more, and to presuppose that such a number exists. No specific condition on context is predicted, except for the fact that such a number must exist, but this is necessarily the case (if none of the women has any children, then this number is 1).[23]

To sum up, while Fox and Hackl (2007) predicts (33) to be felicitous in every context, we predict it to be acceptable only in very specific (and in fact quite implausible) contexts, which we can characterize precisely. If our prediction turns out to be correct, it provides an argument for our approach.

3.2 Negation scoping above both a necessity modal and an indefinite

A similar argument can be made in the case of questions that seem even odder than (33). Suppose that we know that Jack borrowed money from a certain group of people, and therefore owes some money to each of them (not necessarily in the same amounts). Suppose further that Jack is obliged, by law, to reimburse everybody today. Consider the following question in this context:

(41) ?? How much money is Jack not required to give to anyone?

(41) is admittedly very odd in this context. To know what we predict it to presuppose, we should first notice that (41) displays a potential scope ambiguity, depending on whether the NPI *anyone* takes scope under the necessity modal *required* or just above it and below negation. That is, (41) can correspond to the two following (informal) LFs:

(42) a. For what interval I, there isn't a person x such that Jack is required to give to x an amount of money included in I?

[22] See Fox and Hackl's (2007) section 5.2, and in particular p. 568.

[23] Generally speaking, it seems to us that due to this two-step procedure, the universal density of measurement has no consequence whatsoever for the actual interpretation of a degree question once DS has determined it to be grammatical.

b. For what interval I, it is not required that there be someone to whom Jack gives an amount of money included in I?

Given the Maximal Informativity Principle, and on the assumption that it is common knowledge that Jack did not borrow an infinite amount of money, these two LFs are predicted to presuppose the following propositions, respectively:

(43) a. There is an amount of money m such that for every n below m, there is someone to whom Jack is required to give exactly n, and there is nobody to whom Jack is required to give m or more.[24]

b. There is an amount of money m such that for every n below m, it is required that there be someone to whom Jack gives exactly n, and it is not required that there be someone to whom Jack gives m or more.[25]

Both presuppositions are clearly very unlikely to be true, so much so that (41) is expected to be ruled out in nearly all plausible contexts. But let us see what happens if we create a context that happens to satisfy both presuppositions in (43).[26] Of course, we should allow these presuppositions to be satisfied *relatively to a certain standard of granularity*: strictly speaking, if for each value below m, there is someone to whom Jack owes the corresponding amount of money, then there must be infinitely-many people, because there are infinitely-many values below m. From our point of view (cf. subsection 1.2.3), it is permitted to effectively treat the underlying scale as discrete, by rounding any amount expressed in dollars to, say, the closest integer. That is, the presupposition will be satisfied if for each *integer* n below m, Jack owes exactly n dollars to someone. The underlying granularity standard, of course, is itself a contextual matter. Let us now thus consider (42) again (repeated below in (44-b), but against the background given in (44-a):

[24] (42-a) is predicted to presuppose that there is an interval I such that a) there isn't anyone to whom Jack owes an amount of money included in I and b) for every other interval J not included in I, there is someone to whom Jack is required to give an amount of money in J. Assume that this presupposition is satisfied. If I had an upper bound m, then for any amount m' above m, Jack would owe m' to someone, and therefore Jack would owe an infinite amount of money. Assuming that this is excluded by common knowledge, it follows that I has no upper bound, i.e. is of the form $]x, +\infty[$ or $[x, +\infty[$. Hence there must be an amount x such that for any amount strictly below x, there is someone to whom Jack owes that amount.

[25] (42-b) is predicted to presuppose that there is an interval I such that a) it isn't required that there be someone to whom Jack gives an amount of money included in I, and b) for every other interval J not included in I, it is required that there be someone to whom Jack gives an amount of money in J, i.e. for every such J, Jack is required to give an amount of money in J to someone. As before, if it is excluded that Jack owes an infinite amount of money, I must be of the form $]x, +\infty[$ or $[x, +\infty[$. Hence, for the presupposition to be satisfied, there must be an amount x such that for any amount strictly below x, it is required that there be someone to whom Jack gives that amount.

[26] We do not discuss contexts in which only one of the two presuppositions in (43) is satisfied. In practice it is very hard to construct plausible contexts with this property. It is also not clear to us whether both readings triggered by the potential scope ambiguity are in fact possible; when considering only contexts which either satisfy none of the two presuppositions in (43) or both of them, as we did in this section, we do not need engage in this complicated discussion.

(44) a. Many days ago, I don't exactly remember when, Jack started to borrow money from people. One day, he borrowed one dollar from a friend of his. The next day, he borrowed two dollars from another friend. The following day, he borrowed three dollars from yet another friend...and so on. Every day, he borrowed one more dollar than on the previous day from a new person, and he has never reimbursed anyone. I lost track of when exactly all this started, so I don't exactly know what he owes to whom. In any case, today he is required to reimburse everybody. I wonder about the following:

b. How much money is Jack not required to give to anyone?

It seems to us that (41), i.e. (44-b), though hardly understandable in the first context we mentioned (one in which it is only known that Jack borrowed money from various people), becomes significantly more sensical in the context presented in (44-a), as we predict,[27] and is then understood to ask for the smallest amount that Jack doesn't owe to anybody.

To sum up, we predict (41) to be felicitous only in very particular contexts, such as the one presented above, a prediction that seems to be borne out. In contrast with us, Fox and Hackl (2007) predict (41) to be felicitous in every realistic context. Depending on the relative scope of *anyone* and *required*, Fox and Hackl (2007) predicts (41) to presuppose one of the two following propositions:

(45) a. There is minimal amount m such that there isn't anyone in particular to whom Jack is required to give m or more

b. There is a minimal amount m such that it is not required that there be someone to whom Jack gives m or more.[28]

Once evaluated with respect to a realistic granularity standard, both these presuppositions are satisfied as soon as it is known that Jack does not owe an infinite amount of money - which is the case in any realistic context, and, in particular, in the first context we presented.

[27] Note that in this context both presuppositions in (43) are satisfied.

[28] As in the case of (33), Fox and Hackl (2007) predicts (41) to satisfy the Maximal Informativity Principle on the assumption that the Deductive System views the relevant domain of quantification for *anyone* as possibly infinite. In the case where *anyone* takes scope below *required*, Fox and Hackl's (2007) system predicts the Maximal Informativity Principle to be satisfied if for every value x in some interval of real numbers $[k, l[$, it is required that there be someone to whom Jack gives exactly x, and it is not required that there be someone to whom Jack gives l or more. In the case where *anyone* scopes just above *required*, the Maximal Informativity Principle is satisfied if for every value x in some interval of real numbers $[k, l[$, there is someone to whom Jack is required to give exactly x, and there is nobody to whom Jack is required to give l or more. Since (41) passes the test of the Maximal Informativity Principle from the point of view of DS, it can then be interpreted in a 'realistic' context as asking for the smallest amount m (relatively to a contextual standard of granularity) such that it is not required that there be someone to whom Jack gives m or more (or such that there isn't anyone in particular to whom Jack is required to give m or more, depending again on the relative scope of *anyone* and *required*).

4 The Undergeneration Problem

In this section, we will point out that our proposal, in its current form, turns out to predict certain questions to be infelicitous in some contexts where they in fact are felicitous. This can be described as an undergeneration problem; we will have to modify our proposal so as to generate *more* readings than we currently do for certain degree-questions.

In this more sophisticated proposal, we revert to the view that scalar adjectives denote a relation between individuals and degrees, and derive the 'interval-based' semantics by means of a type-shifting operator. This type-shifting operator, noted Π (originally as a mnemonic for *point-to-interval*), has been proposed first in works concerned with the semantics of comparatives (Schwarzschild 2004; Heim 2006). Π applies to any predicate of degrees and turns it into a predicate of intervals; several distinct readings are then predicted for certain degree-questions, depending on the scope of the operator Π.

4.1 Problems

Problem #1.

Imagine a scenario in which we are required to drive between 50mph and 80mph. The interval based account as presented above predicts that the following utterance should express a contradiction:

(46) John knows how fast we are required to drive, but does not know what the maximal permitted speed is.

The reason why we predict this sentence to be contradictory is the following: given the assumption that a sentence of the form *X knows Q*, where *Q* is an interrogative clause, entails *X knows* A_{Q}, where A_{Q} expresses the complete answer to *Q*, the first conjunct of (46) entails that Jack knows the truth of the most informative proposition of the form *We are required to drive at a speed in the interval I*. So if the law determines both a minimal required speed and a maximal permitted speed, then the first conjunct in (46) entails that John knows both the minimal required speed and the maximal permitted speed, which contradicts the second conjunct. This is problematic, since (46) above seems to be fine.

Note that had we used a standard semantics for degree expressions, according to which the question asks for the maximal speed such that we are required to drive at that speed or more (i.e. the minimal required speed), we would in fact correctly expect this example to be logically consistent. On the other hand, we also saw in subsection 2.3 that the stronger reading which we predict must be posited in order to make sense of other cases. The conclusion we can draw is that we must find a way to generate an additional reading on top of the one which our proposal generates in its current form. But then we must also make sure that the results we have achieved so far are not lost.

Problem #2.

The second problem concerns possibility modals in degree questions:

(47) a. How fast are we allowed to drive?
b. For what I, is there a permissible world w (given the regulations in the actual world v) such that our speed is in I in w?

Observe first that for any two intervals I_1, I_2, $I_1 \subseteq I_2$ if and only if *We are allowed to drive in* I_1 entails *We are allowed to drive in* I_2. Now let s be a speed such that it is allowed that our speed be exactly s. The answer based on the interval $[s]$ cannot be entailed by any other true answer, since there can be no interval smaller than $[s]$; therefore, if there is a maximally informative answer, it has to be based on the singular interval $[s]$. But then there can be no other speed s' such that it is allowed that our speed be exactly s' (for if there were such an s', distinct from s, then the answer based on s would not be the maximally informative answer). In other words, s has to be the only speed such that we are allowed to drive exactly at speed s. We conclude that (47) is predicted to presuppose that there is a speed s such that we are *required* to drive exactly at speed s if we drive.

Clearly this is an unwelcome result, since (47) is naturally understood as asking for the *maximal* speed such that we are allowed to drive at that speed, and is felicitous as soon as such a maximal permitted speed exists. This would be predicted by other approaches based on a more standard semantics (Rullmann 1995; Beck and Rullmann 1997), in which (47) would only presuppose that there is a maximal permitted speed.[29] So we need to enrich our proposal so as to generate a similar reading for this case, but without losing our previous results. If we manage do so, (47) will be expected to be ambiguous, and its two different readings will trigger the following presuppositions:

(48) a. **Presupposition predicted by the 'interval-based' semantics**:

[29] Possibility modals in degree questions raise another interesting issue. As pointed out to us by an anonymous reviewer, a question such as (i) below can be understood as asking for an interval such that we are allowed to drive at *any* speed in this interval.

(i) How fast are we allowed to drive?

Under this reading, two people can be said to agree on how fast we are allowed to drive just in case they both know what is the maximal interval such that we are allowed to drive at *any* speed in that interval, i.e. if they know both the minimal and the maximal speed limits. We believe that our interval-based theory can account for this reading provided it is supplemented with a mechanism for deriving so-called 'free-choice effects' – cases where an operator with existential force in the scope of a possibility modal is interpreted as if it were a universal operator taking scope over the modal (for recent proposals, see, e.g., Zimmermann 2000; Schulz 2003; Fox 2007a; Klinedinst 2006; Chemla, to appear). Note that under the 'free-choice' reading, a sentence such as 'we are allowed to drive at a speed between 30*mph* and 80*mph*' ends up meaning that for *any* speed between 30*mph* and 80*mph*, we are allowed to drive at that speed – which is not predicted by standard modal semantics. If the mechanism responsible for free-choice effects can apply to the scope of a degree question operator, then, under our proposal, (i) can be understood as 'For what interval I, is it the case that for any speed s in I, it is allowed that our speed be exactly s?'.

There is a single speed s such that we are allowed to drive exactly at speed s and are not allowed to drive at any other speed.

b. **Presupposition predicted by the 'standard', degree-based semantics**:
There is a speed s such that we are allowed to drive exactly at speed s and are not allowed to drive at any higher speed.

Note that the presupposition in (48-a) logically entails the one in (48-b). It is therefore impossible to find a context where the presupposition of (47) under its 'interval-based' reading is satisfied while the presupposition of (47) under its 'standard', 'degree-based' reading is not. As a result, once we will have enriched our proposal so as to predict two distinct readings for (47), each of which generates one of the presuppositions in (48), we will expect the 'interval-based' reading to be undetectable.[30]

4.2 Solution to the undergeneration problem

In this section, we show that the ambiguity that we need to generate for the above examples can be derived from a variant of a proposal made by Schwarzschild (2004) and Heim (2006) in another context, regarding the semantic analysis of comparative clauses.

In a sense the proposals in Schwarzschild (2004) and Heim (2006) permit us to have our cake and eat it too. They show that we might think of the standard semantics of degree adjectives such as the one in (49) as basic, and derive the interval-based semantics given in (50) via the operator Π defined in (51).[31]

(49) $\|\text{tall}_1\| = \lambda \text{d}.\lambda \text{x.x's height} \geq \text{d}$

(50) $\|\text{tall}_2\| = \lambda \text{D}_{<\text{d,t}>}$: D is an interval.$\lambda \text{x.x's height} \in \text{D}$

(51) $\|\Pi\| = \lambda \text{P}_{<\text{d,t}>}$: P has a maximum. $\lambda \text{I}_{<\text{d,t}>}$: I is an interval. $\max(\text{P}) \in \text{I}$

The operator Π takes two sets of degrees as arguments (the second argument has to be an interval), and presupposes that its first argument has a maximum. When this presupposition is satisfied, it returns the proposition that the maximal degree in the set of degrees that is its first argument is an element of the interval that is its second argument.

(52) $\Pi(\|\text{tall}_1\|) = \lambda \text{I}.\max(\lambda \text{d}.\lambda \text{x.x's height} \geq \text{d}) \in \text{I}$

[30] Note however that if it were known that there is only one permitted speed, the question in (47) would sound rather strange, as one would rather use a necessity modal instead of a possibility modal (*How fast are we required to drive?*). This fact appears to be completely independent of degree-questions proper. Other wh-questions display a comparable behavior: if it is know that Jack is required to read certain books, and is not allowed to read any other book, then a question such as *Which books is Jack allowed to read?* is slightly deviant, as one prefers the contextually equivalent question *Which books is Jack required to read?*. See Magri (2007, 2008), where very similar facts involving declarative sentences are discussed.

[31] We have reversed the order of arguments as compared to Heim's (2006) formulation, but this is harmless.

Let us take an example. The Π operator applied to the expression in (53-a) yields the expression in (53-b), which is equivalent to what we would have gotten had we started with the interval based denotation for the degree adjective in (50):

(53) a. λd.Jack is at least d-fast
b. $\Pi(\lambda$d.Jack is at least d-fast$) = [\lambda$I.Max(λd.Jack is at least d-fast)$\in$ I$] = [\lambda$I.Jack's speed is in I].

Using the Π operator in the case of a simple degree question such as (54) below, we derive the same reading as we did before for (24-b):

(54) a. How tall is Mary?
b. For what interval I, $\Pi(\lambda$d.Mary is d-tall) (I)?
= For what interval I, Mary's height is in I?

However, when the question itself contains an extra operator, such as a necessity modal as in (55), Π can take scope either above or below the modal, which predicts different readings. Thus we predict two possible LFs for the question below:

(55) How fast are we required to drive?
a. For what interval I, it is required that $\Pi(\lambda$d.we are d-fast) (I) = For what interval I, it is required that our speed be in I?
b. For what interval I, $\Pi(\lambda$d.it is required that we drive at least d-fast)(I)?

When Π takes narrow scope, below the modal as in (55-a), the reading we predict is equivalent to the reading that was generated by the first version of our proposal. If however Π takes scope above the modal, as in (55-b), we obtain the reading predicted by Beck and Rullmann (1997) and Fox and Hackl (2007). This is so because the expression *λd.it is required that we drive at least d-fast* denotes the set of intervals that include the highest speed s such that we are required to drive at least s-fast. As a result, (55), under this reading, asks for the most informative answer of the form 'The maximal speed s such that we are required to drive s-fast and are not required to drive faster is in the interval I'. This maximally informative answer will be based on the smallest possible interval yielding a true answer, i.e. the unique speed s such that we are required to drive s-fast or more and are not required to drive faster, i.e. the minimal required speed. For instance, if we are required to drive between $45mph$ and $75mph$ on the highway, the maximal speed s such that we are required to drive s-fast or more is $45mph$ and therefore '$\Pi(\lambda$d.it is required that we drive at least d-fast)' denotes all the intervals that include $45mph$. In this context, the maximally informative answer to (55), parsed as in (55-b) is the proposition based on the singleton interval $\{45mph\}$, i.e. states that the minimal required speed is $45mph$. But if (55) is understood as in (55-a), then the maximally informative answer has to specify not only that the minimal required speed is $45mph$, but also that the maximal permitted speed is $75mph$.

We are now in a position to address the problem with examples (46) and (47) noted above. The reason why (46), repeated below as (56), is not contradictory is that in the LF of the embedded question Π operator now can take narrow or wide scope. If it takes wide scope, the predicted reading is equivalent to what the 'standard', degree-based semantics, generates, as we have just seen; no contradiction is produced, since the first conjunct now simply asserts that John knows what the minimal required speed is, and does not entail that he knows what the maximal permitted speed is.

(56) John knows how fast we are required to drive, but does not know what the maximal permitted speed is.

In contrast, in the case of (32-b) in subsection 2.3, repeated below as (57), Π in the embedded question has to take narrow scope in order for the whole discourse to be coherent.

(57) a. Jack and Peter are devising the perfect Republic. They argue about speed limits on highways. Jack believes that people should be required to drive at a speed between 50mph and 70mph. Peter believes that they should be required to drive at a speed between 50mph and 80mph. Therefore ...
b. Jack and Peter do not agree on how fast people should be required to drive on highways.

Finally, in (47), repeated below as (58), Π can again either take wide or narrow scope with respect to the modal.

(58) How fast are we allowed to drive?
a. For what interval I, Π (λd.we are allowed to drive at least d-fast)(I)?
b. For what interval I, it is allowed that Π (λd.we drive d-fast)(I)?

The reading corresponding to the LF in (58-b) is the one that was predicted before we introduced Π, i.e. presupposes that there is a single speed *s* such that we are allowed to drive at that speed and are not allowed to drive at any other speed (cf. subsection 4.1). The reading corresponding to the LF in (58-a) is the one predicted by degree-based approaches (Rullmann 1995; Beck and Rullmann 1997; Fox and Hackl 2007), and only presupposes that there be a speed *s* such that we are allowed to drive at *s* and are not allowed to drive at any higher speed.[32]

4.3 Negative islands revisited

Our proposal now predicts degree-questions to be potentially ambiguous, depending on the scope of Π. It is thus important to check that our previous

[32] As we'll see in subsection 4.5, Heim's (2006) use of Π was also meant to capture certain ambiguities, in an other but related domain, that of comparative clauses that contain a modal or a quantifier.

account of negative islands is not lost, i.e. that our amended proposal does not introduce a new reading for a question such as (3-b) which might be able to satisfy the Maximal Informativity Principle.

Let us thus go back to negative island cases. A question such as (59-a) below now can correspond to two distinct logical forms, paraphrased in (59-b) and (59-c):

(59) a. # How fast didn't Mary drive?
b. For what interval I, $\Pi(\lambda$d.Mary didn't drive d-fast)(I)?
c. For what interval I, it is not the case that $\Pi(\lambda$d.Mary drove d-fast)(I)? = For what interval I, Mary's speed is not in I?

The LF in (59-c) is equivalent to the one given in (25-b), in section 2.2.2. We have already shown in this section that this LF is excluded by the Maximal Informativity Principle. As to the LF in (59-b), note that *λd.Mary didn't drive d-fast* denotes the (characteristic function of) the set of degrees higher than Mary's speed, and therefore does not contain a maximum. But the operator Π is defined in terms of the *maximum* of the degree-predicate is applied to. As a result, the semantic value of Π*(λd. Mary didn't drive d-fast)* is not defined, i.e. does not denote anything, and therefore (59-b) itself does not have a semantic value (another way to put things is to say that the question has a contradictory presupposition, namely that the predicate *λd.Mary didn't drive d-fast* has a maximum).

More generally, degree questions in which Π takes maximal scope will follow the pattern originally predicted by Rullmann (1995) for all degree questions: whenever a derived degree predicate is upward-scalar, in the sense given in (14), Π cannot apply to it. As a result, Π cannot scope over a DE-operator which takes immediate scope over a lexical scalar predicate (because when a DE-operator scopes over a lexical scalar predicate, abstracting over the degree argument of the scalar predicate results in an upward-scalar predicate). [33]

It follows that when a DE-operator intervenes between the *how*-phrase and a scalar predicate, Π must take scope below the DE-operator, and therefore such cases are expected to behave just as predicted by the more simple version of our proposal presented in section 2. Our previous discussions of negative islands and *quasi* negative islands thus remain unaffected.

[33] Because we define Π in terms of maximality, our final proposal clearly bears some resemblance with Rullmann's (1995) proposal. But while Rullmann's maximality operator necessarily takes wide-scope, the scope of Π with respect to various operators is free. An anonymous reviewer wonders whether this difference alone, coupled with the Maximal Informativity Principle, might be sufficient to explain all the data, without making use of intervals. More specifically, the reviewer's suggestion is that one could account for all the basic facts by maintaining a standard semantics (i.e. one in which the variable bound by *how* ranges over degrees rather than over intervals of degrees) and letting a Rullmannian maximality operator take narrow scope. While such a proposal would be able to predict basic negative island effects and the possibility of modal obviation, it would fail to predict the right readings both for cases of modal obviation ((26)) and for the cases discussed in sections 2.3, 3 and 4.5.

4.4 Truncation: resetting the lower bound of the scale

Consider again the following (paradigmatic) example:

(60) How fast are we not allowed to drive on this highway?

In principle, there are three different possible insertion sites for Π, corresponding to the following three semi-formal representations:

(61) a. For what I, Π [λd. [we are not allowed to drive d-fast]] (I)?
b. For what I, not [Π [λd. [we are allowed to drive d-fast]](I)]?
c. For what I, it is not allowed that [Π [λd. [we drive d-fast]](I)]?

Now, (61-a) is ruled out by the fact that Π cannot scope immediately over negation, as explained in subsection 4.3. (61-b) is ruled out by the Maximal Informativity Principle for the very same reason why simple negative degree questions are ruled out.[34] So the only possible LF is the one in (61-c), which corresponds to what our initial interval semantics delivered:

(62) For what interval I, it is not allowed that our speed is included in I?

This means that it should not be possible to use (60) felicitously in a context where it is known that there is both a maximal permitted speed and a minimal required speed, as in this case the set of degrees for which it is not allowed that our speed be included in that set does not form an interval. However, Danny Fox (p.c.) points out that contrary to our prediction, (60) can be used in such a context, and is then understood as asking for the maximal permitted speed.

Here is our proposed solution to this problem. We make the additional assumption that a degree question can be interpreted relatively to a contextually *truncated* scale, i.e. with respect to a scale from which an initial segment of the lexically determined scale (in this case the scale of speeds) has been removed. More specifically, we adopt a modified version of a proposal put forward by Rett (2007), in another context. Rett introduces an operator, $EVAL$, which denotes a function form sets of degrees to sets of degrees, and which, when applied to a set of degree D, returns the subset D' of D consisting of all the members of D which are higher than a certain contextually determined degree. $EVAL$ is then, in a sense, a 'truncation' operator.[35] Our only innovation with respect to Rett's proposal is that we will define our truncation operator in presuppositional terms. In Rett's framework, a predicate such as $EVAL(\lambda d.\phi(d))$, when

[34] Generally, the Maximal Informativity Principle prevents any LF of the form $[How]_I[NOT[\Pi[\lambda d.\phi(d)]](I)]$ from being felicitous when ϕ denotes a downward-scalar predicate of degrees, however complex ϕ is.

[35] The motivation for this proposal is to account for the evaluative reading of 'short' in *Jack is as short as Mary*, which presupposes that Mary is short. Rett (2007) derives this presupposition by ensuring that $EVAL$'s presence is obligatory when the marked member of a pair of antonyms occur in certain environments, due to some general pragmatic considerations. According to Rett, $EVAL$ can also optionally combine with 'positive' adjectives like 'tall'. She also points out in passing that a question such as 'How short is John?' presupposes that John is short, which, in her terms, means that $EVAL$'s presence is compulsory in this construction as well.

applied to a degree of height which is below the relevant contextually determined degree s, returns the truth-value $FALSE$. Contrary to Rett, we will define the 'truncation operator', noted $\mathcal{T}$, in such a way that the result of applying $\mathcal{T}(\lambda d.\phi(d))$ to a degree which is below the threshold s is *undefined*. In other words, $\mathcal{T}(\lambda d.\phi(d))$ is a *partialization* of ϕ, i.e. $\mathcal{T}$ turns a function that is defined for a given scale into a function that is defined only for a proper subpart of the very same scale (the segment above s), and is otherwise identical. Further, we define $\mathcal{T}$ as combining directly with predicates of type $< d, < e, t >>$ and as returning predicates of the same type:[36]

(63) $\|\mathcal{T}\| = \lambda P_{<d,<e,t>>}.\lambda x_d : x > s.\lambda y_e.P(x)(y)$

Now, recall that we view the variable introduced by 'how' as ranging over intervals of degrees *in the scale associated with the scalar predicate that 'how' combines with*. The scale associated with a scalar predicate can be identified with the set of values for which the scalar predicate is defined. Thus, if 'how' combines with $\mathcal{T}(fast)$, it will bind a variable ranging over intervals of degrees for which $\mathcal{T}(fast)$ is defined, i.e. intervals of degrees of speed that are higher than the contextually determined speed s. Once this is in place, a possible LF representation for (60) is the following:

(64) For what interval of degrees I in the scale of $\mathcal{T}(fast)$, it is not allowed that $[\Pi[\lambda d.[$ we drive $d - [\mathcal{T}(fast)]]](I)]$?

Suppose the contextual standard is s. Then (64) can be paraphrased as follows:

(65) For what intervals I of degrees of speed above s, it is not allowed that the maximal degree d such that we drive $d - \mathcal{T}(fast)$ belongs to I?

Suppose that there are both a minimal required speed d_1 and a maximal permitted speed d_2. If the contextually truncated scale does not include d_1, i.e. if $s > d_1$, then the Maximal Informativity Principle will be satisfied. Namely, relatively to this truncated scale S, there is an interval, i.e. $]d_2, +\infty[$, such that we are not allowed to drive in this interval and we are allowed to drive at any speed *in the truncated scale* below this interval. The presupposition induced by the Maximal Informativity Principle is then the proposition that there is a speed t such that we are not allowed to drive at speed t or more, and we are allowed to drive at any lower speed *in the contextually truncated scale* - which does not exclude that there be a minimal required speed *below* the point of truncation, i.e. below s.

One more remark is necessary. The possibility of using truncated scales clearly represents a liberalization of the constraints imposed by the Maximal Informativity Principle. So we have to check that the resulting theory is not *too*

[36] We use Heim and Kratzer's (1998) notation for representing partial functions. That is, if x is a variable of arbitrary type, $\lambda x : \psi(x).\phi(x)$ denotes the partial function which is defined only for the objects e which are in the extension of ψ and such that, when applied to an object e which is in the extension of ψ, returns $TRUE$ if e is in the extension of ϕ, and $FALSE$ otherwise.

liberal, i.e. does not predict bad cases of negative islands to be felicitous. Let us thus consider again a paradigmatic case of negative island:

(66) *How fast didn't John drive ?

Suppose (66) is evaluated with respect to a truncated scale of the form $]s, +\infty[$. This amounts to saying that (66)'s logical form is the following, where s serves as the threshold for $\mathcal{T}$:

(67) For what interval I in the scale of $\mathcal{T}(fast)$, $\neg[\Pi[\lambda d.[$ John drove $d - [\mathcal{T}(fast)]]]](I)]$?

Now, note that $[\Pi[\lambda d.[$John drove $d - [\mathcal{T}(fast)]]]$ presupposes that $[\lambda d.[$John drove $d - [\mathcal{T}(fast)]]]$ has a maximum, and denotes the set of intervals that contains this maximum. But $[\lambda d.[$John drove $d - [\mathcal{T}(fast)]]]$ has a maximum if and only if Jack drove at a speed above s.[37] Hence, $[\Pi[\lambda d.[$John drove $d - [\mathcal{T}(fast)]]]$ triggers the presupposition that Jack's speed was above s, and this presupposition is inherited by $\neg[\Pi[\lambda d.[$John drove $d - [\mathcal{T}(fast)]]]](I)]$. Given reasonable assumptions about the way presuppositions project in questions, this presupposition will also be inherited by the question as a whole. In other words, (67) presupposes that Jack drove above the contextually determined speed s. (67) can be informally paraphrased as follows:

(68) For what interval I of degrees of speed above s, John's speed did not belong to I.

Given that it is presupposed that John's speed was above s, there cannot be a maximally informative answer to (68), for exactly the same reason as before. Let d be John's speed. Either d is the unique minimal degree in $]s, +\infty[$, if there is such a minimal degree (i.e. if the underlying scale is conceptualized as discrete), or d is not. Suppose that it is not. Then every answer based either on an interval included in $]s, d[$ or in $]d, +\infty[$ is a true answer; but then no true answer entails all the true answers, and the Maximal Informativity Principle is not met. Hence, the MIP can be met only if in every world compatible with the common ground, John's speed is the minimal speed in $]s, +\infty[$ (assuming there is such a minimal speed). However, such a context would not in fact satisfy the Maximal Informativity Principle, for it would be a context in which it is already known what the maximally informative answer is (since it would be the same one in every world compatible with the common ground).

We can thus conclude that even in a theory where scales can be truncated, (68) is still ruled out by the Maximal Informativity Principle.[38]

[37] Suppose that Jack did not drive at a speed above s. Then for every degree of speed d, either 'John drove $d - \mathcal{T}(fast)$' is false (if $d > s$), or 'John drove $d - \mathcal{T}(fast)$' does not have a truth-value (if $d \leq s$). So the set of degrees d such that 'John drove $d - \mathcal{T}(fast)$' is true is the empty set, which has no maximum.

[38] Using truncated scales also enables us to address the problem mentioned in footnote 19.

4.5 Restrictions on the scope of Π

Imagine that you are driving very fast on the highway, say at 100 mph, when a policeman stops you. In such a context, it is clear that you are probably driving too fast, and certainly not too slow. You pretend to be surprised and ask:

(69) Why did you stop me,
- a. How fast should I have driven ?
- b. How fast am I supposed to drive ?
- c. # How fast am I required to drive? (Giorgio Magri, p.c.)

It seems that in the above scenario it is possible to utter (69-a) or (69-b), which can be naturally understood to ask for the range of speeds such that it would have been reasonable to drive at a speed in this range. These questions, in this context, can therefore be used to express an uncertainty as to what the maximal permitted speed is. This is in accordance with what our analysis predicts: on the reading where Π scopes below the necessity modal, the true answers to these questions are based on the intervals I such that you must drive at a speed in I. So the maximally informative answer will necessarily state what the maximal permitted speed is, if there is one.[39] A puzzling fact, from our perspective, is that (69-c) is infelicitous in the above context: it clearly suggests that the speaker believes he was not driving fast enough. In other words, (69-c) is understood to ask for the minimal required speed, which makes it infelicitous in such a context, where it is clear that the minimal required speed is not at issue.

So far, our proposal does not predict this contrast.[40] Recall that we predict two scope possibilities for Π in the above questions: it can scope above the necessity modal, in which case we generate a reading which is equivalent to what the 'standard' view of degree questions would predict, (70-a), or it can scope below the modal, and give rise to the interval-based reading,(70-b).[41]

(70)
- a. For what I, [Π [λd. [$\Box$ I drive d-fast]]] (I)?
  $\rightsquigarrow$ For what interval I, I contains the speed s such that I are must drive s-fast and am not required to drive faster than s?
  = What is the minimal required speed?
- b. For what I, $\Box$ [Π [λd. [I drive d-fast]](I)]?
  $\rightsquigarrow$ For what interval I, for every permissible world w, my speed in w belongs to I?

The restriction we observe in (69), then, is that Π seems to prefer to take scope below *should* or *supposed to* and it does not like to scope below *require*.

[39] And also, in principle, what the minimal required speed is, if there is one. Note, however, that if the scale is truncated (cf. subsection 4.4), then the minimal required speed might fall outside of it and hence not be relevant at all.

[40] Proposals such as Beck and Rullmann (1997, 1999); Fox and Hackl (2007), fail to capture this contrast as well: they predict that each of the three questions in (69) presupposes that there was a minimal required speed.

[41] the sign '$\Box$' stands for any necessity modal.

Interestingly, the fact that we get different interpretations for degree questions depending on which necessity modals occur in them is reminiscent of phenomena that have been observed in another, related domain, namely the domain of comparatives. Thus consider the sentences in (71) below, in a context where there is both a minimal required speed and a maximal permitted speed. (71-a) and (71-b) suggest that John drove too fast, i.e. faster than the maximal permitted speed. (71-c) however merely suggests that John drove faster than the minimal required speed, i.e. 50 mph, and does not imply that John violated the regulations. Thus while (71-a) and (71-b) force, or at least strongly favor, the reading according to which John drove too fast (the 'higher-than-max' reading), (71-c) favors a weaker reading (the 'higher-than-min' reading) and it could be paraphrased as 'John drove faster than the minimal required speed'.[42]

(71) a. John drove faster than he should have driven
b. John drove faster than he was supposed to.
c. John drove faster than he was required to drive.

This parallelism between degree questions and comparatives suggests that the same mechanism is at play in both cases. That is, whatever mechanism is used in order to explain the existence and distribution of the higher-than-min and higher-than-max readings in comparatives should also be used in order to characterize the semantics of degree questions. In the framework of Schwarzschild (2004) and Heim (2006), the difference between the 'higher-than-min' and the 'higher-than-max' readings is reducible to a difference in the relative scopes of the modal operator and the Π operator, which is assumed to be always present in a than-clause. In their system, the fact that (71-c) prefers the higher-than-min reading follows from a preference for Π to take scope above *require*, while the fact that (71-a) and (71-b) favor the higher-than-max reading follows from a preference for Π to take scope below *should have, be supposed to*.[43] To sum up, the restrictions on the scope of Π in degree questions which we need to assume in relation to the examples in (69) seem to be identical to the ones that need to be posited by in the case of comparatives, and this provides further support for our general approach.

5 Conclusion

In this paper, we have argued for a new approach to the semantics of degree questions, according to which the LF of a degree question contains a variable that ranges over intervals of degrees, by analogy with Schwarzschild and Wilkinson's (2002) proposal regarding the semantics of comparative clauses. We have

[42](71-c) might nevertheless be ambiguous between the weaker reading and the 'higher-than-max' reading. What is important for us is that 'be required' clearly behaves differently from 'be supposed to' and 'should have' in favoring the 'higher-than-min' reading. Thanks to Vincent Homer for discussion.

[43]Note however that in the case of a question such as 'How fast should people be required to drive on highways?', we must allow Π to scope below *required* in order to predict the interval-based reading discussed in section 2.3 (cf. example (32)).

showed that not only is there independent evidence for the existence of the interval-based reading of degree questions, but also that this assumption, together with the Maximal Informativity Principle, can correctly predict the classic negative island facts, as well as the cases of modal obviation discussed by Fox and Hackl (2007). Further, we have argued that our proposal makes correct predictions regarding the context sensitivity of certain cases that we called quasi-negative islands - degree questions which are unacceptable out of the blue, but which improve significantly in very specific contexts. The last section of our paper provided a more sophisticated version of the basic proposal, which allowed us to derive certain additional readings without losing our previous results. Finally, we showed that some subtle ambiguities involving modals in comparative clauses are also present in degree questions, which calls for a unified theory. We hope to have shown that an interval-based semantics for both comparatives and degree questions can serve as the basis for such a theory.

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References

Abrusán, M. (2007). *Contradiction and Grammar: the Case of Weak Islands.* Ph. D. thesis, M.I.T.

Abrusán, M. (2008). A Semantic Analysis of Negative Islands with Manner Questions. In A. Grøn (Ed.), *Proceedings of Sinn und Bedeutung 12.*

Abrusán, M. and B. Spector (2008). An interval-based semantics for degree questions: Negative islands and their obviation. In N. Abner and J. Bishop (Eds.), *Proceedings of the 27th West Coast Conference on Formal Linguistics*, Somerville, MA, pp. 17–26. Cascadilla Proceedings Project.

Beck, S. and H. Rullmann (1997). Degree Questions, Maximal Informativeness and Exhaustivity. In P. Dekker and M. Stockhof (Eds.), *Proceedings of the Tenth Amsterdam Colloquium*, pp. 73–92.

Beck, S. and H. Rullmann (1999). A Flexible Approach to Exhaustivity in Questions. *Natural Language Semantics 7*, 249–298.

Bierwisch, M. (1989). The semantics of gradation. In M. Bierwisch and E. Lang (Eds.), *Dimensional Adjectives: Grammatical Structure and Conceptual Interpretation*, pp. 71–261. Springer-Verlag Berlin and Heidelberg GmbH & Co. K.

Chemla, E. (To appear). Similarity: Towards a unified account of scalar implicatures, free choice permission and presupposition projection. To appear in *Semantics and Pragmatics*.

Dayal, V. (1996). *Locality in WH quantification: Questions and Relative Clauses in Hindi*. Studies in Linguistics and Philosophy. Kluwer Academic Publishers.

Fox, D. (1999). Reconstruction, binding theory, and the interpretation of chains. *Linguistic Inquiry 30*, 157–186.

Fox, D. (2007a). Free choice and the theory of scalar implicatures. In P. Stateva and U. Sauerland (Eds.), *Presupposition and Implicature in Compositional Semantics*. Palgrave-Macmillan.

Fox, D. (2007b). Too many alternatives: Density, symmetry, and other predicaments. In *Proceedings of Semantics and Linguistic Theory 17*, Volume 17.

Fox, D. and M. Hackl (2007). The Universal Density of Measurement. *Linguistics and Philosophy 29*, 537–586.

Gajewski, J. (2002). On Analyticity in Natural Language. Ms., MIT.

Heim, I. (2006). Remarks on comparative clauses as generalized quantifiers. Ms., M.I.T, available at `http://semanticsarchive.net/Archive/mJiMDBlN`.

Heim, I. and A. Kratzer (1998). Semantics in the Generative Grammar. Malden.

Heycock, C. (1995). Asymmetries in reconstruction. *Linguistic Inquiry 26*, 547–570.

Jacobson, P. (1995). On the Quantificational Force of English Free Relatives. In E. Bach (Ed.), *Quantification in Natural Languages*, pp. 451–486. Kluwer, Dordrecht.

Kennedy, C. (1999). *Projecting the adjective: The syntax and semantics of gradability and comparison*. Garland, New York.

Kennedy, C. (2001). Polar Opposition and the Ontology of 'Degreesí. *Linguistics and Philosophy 24*(1), 33–70.

Klinedinst, N. (2006). *Plurality and Possibility*. Ph. D. thesis, UCLA.

Kroch, A. (1989). Amount quantification, referentiality, and long Wh-Movement. Ms., University of Pennsylvania.

Kuno, S. and K. Takami (1997). Remarks on negative islands. *Linguistic inquiry 28*(4), 553–576.

Magri, G. (2007). A theory of individual level predicates based on blind scalar implicatures. Ms. MIT.

Magri, G. (2008). Mismatching Scalar Implicatures. Draft version available at `http://web.mit.edu/gmagri/www/papers/MITWPLpaper.pdf`, to appear in a forthcoming issue of *MIT Working Papers in Linguistics*.

Rett, J. (2007). Antonymy and Evaluativity. In *Proceedings of SALT 17*.

Rizzi, L. (1990). *Relativized Minimality*. MIT Press.

Rullmann, H. (1995). *Maximality in the Semantics of Wh-Constructions*. Ph. D. thesis, University of Massachusetts, Amherst.

Schulz, K. (2003). You may read it now or later. A case study on the paradox of free choice permission. Master's thesis, University of Amsterdam.

Schwarzschild, R. (2004). Scope-splitting in the comparative. Handout from MIT colloquium, available at `http://www.rci.rutgers.edu/~tapuz/MIT04.pdf`.

Schwarzschild, R. and K. Wilkinson (2002). Quantifiers in comparatives: A semantics of degree based on intervals. *Natural Language Semantics 10*(1), 1–41.

Seuren, P. (1984). The Comparative Revisited. *Journal of Semantics 3*(1-2), 109–141.

Sharvit, Y. (2002). Embedded Questions and *de dicto* readings. *Natural Language Semantics 10*, 97–123.

von Stechow, A. (1984). Comparing Semantic Theories of Comparison. *Journal of Semantics 3*(1-2), 1–92.

Szabolcsi, A. and F. Zwarts (1993). Weak islands and an algebraic semantics for scope taking. *Natural Language Semantics 1*, 235–284.

Zimmermann, T. E. (2000). Free Choice Disjunction and Epistemic Possibility. *Natural Language Semantics 8*(4), 255–290.